Lesson 6: Manipulating Equations

Manipulating equations is probably one of the most important skills to master in a high school physics course.

- Although it is based on familiar (and fairly simple) math concepts, it is still a stumbling block for most new physics students.
- Manipulating an equation means that you rearrange the equation so that the unknown you are trying to calculate is on its own on one side of the equation.
- Later in the course it will also give you the power to combine formulas (which is necessary) to solve more complicated problems.
- Learning how to manipulate formulas now (while the formulas are still easy ones!) will pay off later.

At all times remember two basic rules from math...

1. **To move something to the other side, just do the opposite math operation to it.**
2. **If you do it to one side, do it to the other.**

**Example 1:** The basic formula for calculating the velocity of an object is \( v = \frac{d}{t} \), where "v" is the velocity, "d" is the displacement, and "t" is the time. This formula is great "as-is" if we are going to calculate velocity, but what if I need to calculate the displacement and I've been given the velocity and time? **Solve** the formula to solve for "d".

In the formula \( v = \frac{d}{t} \), "d" is being divided by "t". **To get "t" to the other side, we need to do the opposite... multiply by "t"!**

\[
\begin{align*}
  v &= \frac{d}{t} \\
  v &= \frac{d}{t} (t)
\end{align*}
\]

**but what we do to one side, we do to the other...**

\[
(t) v = \frac{d}{t} (t)
\]

the "t" on the right side cancel each other out leaving...

\[
(t) v = d
\]

the last step (and it's basically just a tradition) is to put our unknown on the left side of the equation. So let's flip flop the whole thing to get our final equation!

\[
d = t v
\]

And that's it! This is not a new formula you need to memorize; it is a formula already on your data sheet that you have manipulated to use more easily for a particular question.
Now try to solve the same formula for "t". You should get...

\[ t = \frac{d}{v} \]

Be careful with formulas with addition, subtraction, square roots and squares.

- You basically need to follow the **BEDMAS** (Brackets, Exponents, Division, Multiplication, Addition, Subtraction) rule from math, but backwards.
- Usually take care of any addition and subtraction first, then multiplication and division, and finally exponents (remember, square root is just an exponent).

**Example 2: Solve** the formula “\( v_f^2 = v_i^2 + 2ad \)” for \( v_i \)

Before doing anything else, take care of anything being added or subtracted to \( v_i \) by doing the opposite...

\[ v_f^2 - (2ad) = v_i^2 + 2ad - (2ad) \]

which leaves us with...

\[ v_f^2 - 2ad = v_i^2 \]

flip the whole formula (so \( v_i \) is on the left) and take the square root of both sides...

\[ v_i = \sqrt{v_f^2 - 2ad} \]

and you're done!