Lesson 14: Vectors in Two Dimensions

Two dimensional problems are a little tougher, because we are no longer just lining up collinear vectors and doing quick math.
- Instead, we need to pay attention to how 2D vectors form a more complex (but not very complex) **non-collinear** diagram. Mostly this will involve right angle triangles.
  - If they are right angle triangles, just use your regular trig (SOH CAH TOA) and pythagoras ($c^2 = a^2 + b^2$).
- You'll want to be thinking about *physics* as you set up your diagram (so that you get everything pointing head-to-tail and stuff) and then switch over to doing *math* just like any trig problem.
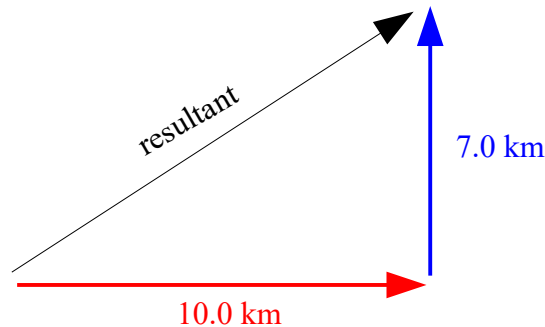
Components of Vectors

One of the most important ideas in vectors is components.
- Just like a component stereo system is made of several individual parts working together, **components of vectors** are the individual parts that add up to the overall **resultant**.

Example 1: A car drives **10.0 km [E]** and then **7.0 km [N]**. **Determine** its displacement.
　　First, draw a proper diagram...

Illustration 1: Car moving.

> **Warning!**
> To truly be called components, the two vectors must be perpendicular to each other. Usually this is tied back in to math by referring to them as the x-component and y-component.

The **red** and the **blue** vectors are the components of the resultant.
- The **red** and **blue** components show you how walking **East** and the **North** will result in you moving more or less in a North-East direction.

Notice how this diagram even shows the vectors being added in the correct order according to the question.
- **10 km [E]** is shown leading up to **7.0 km [N]**. Start at the tail of the red arrow and follow the path it takes you along. You eventually end up at the head of the blue vector.
- If you added them with **7.0 km [N]** and then **10 km [E]** you would still get the same final answer, just with a different angle because of a different reference point.
- The resultant is drawn in head-to-head and tail-to-tail, just like a resultant is always supposed to be.

This is certainly a right angle triangle, so just use $c^2 = a^2 + b^2$ to find out the magnitude (*size*) of the resultant.

$$c^2 = a^2 + b^2$$
$$c^2 = 10^2 + 7.0^2$$
$$c = 12.206556 = 12 \, km$$

Since this problem is about vectors we would also have to give a direction in our answer.

- This will involve measuring an angle, but it will also involve a way to make sure everyone knows what that angle means.
- We will leave that part of the problem until after we have finished the next section...

Vector Directions

The problem with angles is that on their own they don't mean very much.

- If I say to you "35°", you need to ask "35° pointing where?"
- Over the years there have basically been two systems that are accepted for giving meaning to the angles used in Physics:
    1. **Cartesian Method**
    2. **Navigator Method**
- You are expected to understand and be able to use both.

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Did You Know?

The Cartesian Method is named after the mathematician and philosopher René Descartes, who created the system around 1637.

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Cartesian Method

Although you might not always refer to it using the name "Cartesian", you have almost certainly used this system in math at some time. It is basically the "x-axis" system you learned in Science 10.

- It divides space into four quadrants using an x and y axis.
- The positive x axis is considered to be the beginning and is assigned an angle of 0°.
- As you move counter-clockwise, the angles get bigger. At each axis point you have added another 90°.

The Cartesian method is fine for math class, but it does have one serious drawback; the positive x axis being set as 0° is an arbitrary (*for no good reason*) decision.

- For this reason I will not use this method in the notes as often. That does **not** mean that you can ignore it.
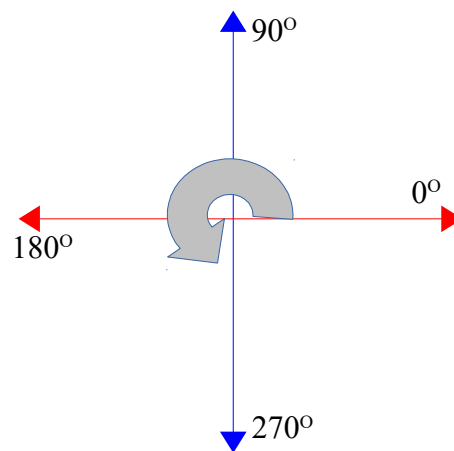
Illustration 2: Cartesian Method

Navigator Method

The navigator method gets rid of the arbitrary nature of the Cartesian method by using compass points.

- We will use one of the four compass points as our reference, and then measure how many degrees towards one of the other compass points we have moved.
- We do have to be a little careful, because when using the navigator method there are two different styles of writing down the answer, and every answer can be given using two different angles!
  - The following examples will show you how this can work.
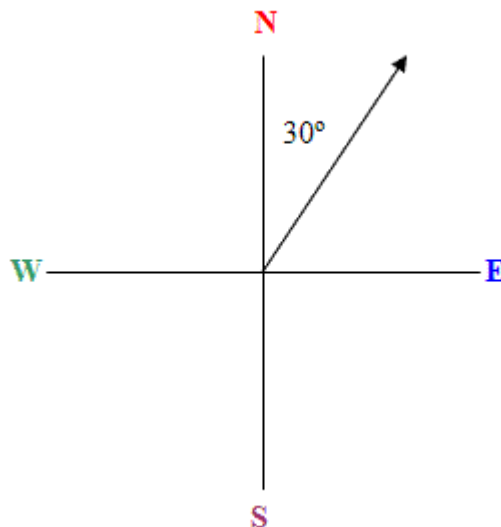
Example 2: **Identify** the direction of the vector shown in *Illustration 3*.

Illustration 3: Navigator method example.

If you look at where the angle is placed in this diagram, you'll probably agree that we are measuring an angle that is 30° away from the **North**.

- It's like we are using **North** as a reference line in this situation. The vector is 30° away from **North**.

Looking at the diagram, we're basically going clockwise towards **East** a little bit.

- So, we know that we are using **North** as a direction, but we're leaning over towards **East** somewhat.

In the physics style of giving a direction, we would write [**N**30°**E**], which is read as "**North** 30° **East"**. The reference line [**North**] is given first, and then the number of degrees away from it, 30°, going towards another of the reference lines [**East**].

We always put directions in **[square brackets]** to show that they are the direction, and not some weird formula!

Since all the way from North to East is a full 90°, we know that this could also be drawn showing that the vector is 60° (we get it from 90° - 30°) *counterclockwise* from the ***East***.

- This would mean that we could also measure this vector as [**E**60°**N**] .
- Between these two ways of measuring, [**N**30°**E**] is considered more, ummm, *polite*, since the angle is smaller. Either one is still correct.

Your text book has chosen to use a slightly modified (and I don't like it much) style of using the Navigator method.

- In this system the order is changed to (1) the number of degrees, (2) towards a reference, (3) from a reference.
- For example, the text book would say that the direction in Illustration 3 is [30° **E** of **N**]. Notice how it says you are 30 degrees towards the **East** from **North**. In fact, we would read it as "30 degrees **East** of **North**."
- I will avoid using this style in the notes, but you must be aware of it and be able to use it, especially when doing questions in the text book.

Example 3: At the beginning of this lesson in **Example 1**, you figured out the distance a car moved, but didn't give its direction. To give an answer of his displacement (a vector measurement) you need to **determine** the direction of the vector.

As a hint, you should probably use tan to figure out this angle. Using either sine or cos will involve using the resultant you just calculated. If you got the resultant wrong, you'd get your angle wrong also. Traditionally you measure from the start of the resultant, at its tail. This means we will measure the angle at the bottom left of the diagram.

$$\tan \theta = \frac{opp}{adj} = \frac{7.0}{10.0}$$
$$\theta = 34.99202 = 35^o$$

This means that the final answer for the question is that the displacement of the person is **12 km [E35ºN].**

Homework

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