Lesson 11: The Other Formulas

There are several other formulas that are very useful when the acceleration is uniform.

● Do not use these equations if the acceleration is changing! The acceleration must be constant.
● These other formulas are based on combinations of the basic velocity and acceleration formulas, as well as interpreting graphs.
● Although the formulas are shown here in a particular numbered order, you do not need to identify them this way.

Formula 1

The first formula is based on knowing information about displacement, final and initial velocities, and time.

\[ d = \left( \frac{v_f + v_i}{2} \right) t \]

● At first this may seem to be an odd acceleration formula, since acceleration does not appear in the formula as a variable.
  ● Notice that there are two velocities, \( v_f \) and \( v_i \), so we know that there must be acceleration.
  ● Only use this formula when you know for certain that the object has been going through a constant acceleration, even though “a” doesn’t appear in the formula.

Example 1: Determine how far a vehicle moved if it started at 12 m/s and accelerated up to 28 m/s in a time of 34s.

\[ d = \left( \frac{v_f + v_i}{2} \right) t \]
\[ d = \left( \frac{28 + 12}{2} \right) 34 \]
\[ d = 680 = 6.8 \text{e}2 \text{ m} \]

Formula 2

We will do problems where we have information about displacement, initial velocity, time, and acceleration. The formula for these situations is...

\[ d = v_i t + \frac{1}{2} a t^2 \]

● Be careful with this formula. Only the time is squared in the last term, not acceleration and time.
● As a bonus, a lot of the time \( v_i \) will be zero, which cancels out the first term and leaves you with...

\[ d = \frac{1}{2} a t^2 \]
Example 2: Occasionally the US Air Force calls me in to test fly their “birds”. A few weeks back I was flying along in my F-22 at 97m/s when I decide to kick in the afterburners for 12.3s. If the afterburners can generate enough thrust to accelerate the F-22 at 26m/s², determine how far I travelled during that time.

\[ d = v_i t + \frac{1}{2} a t^2 \]
\[ = (97) (12.3) + (0.5) (26) (12.3)^2 \]
\[ d = 3159.87 = 3.2e3 \text{ m} \]

Example 3: I am in a F-22 that is on the runway. From rest, I accelerate the plane at 3.9m/s² for 14.5s to reach take off velocity. Determine how long the runway must be.

This is an example of a question where the initial velocity is zero (since I’m starting from rest), so...

\[ d = v_i t + \frac{1}{2} a t^2 \]
\[ d = \frac{1}{2} a t^2 \]
\[ = (0.5) (3.9) (14.5)^2 \]
\[ d = 409.9875 = 4.1e2 \text{ m} \]

Formula 3
There is a formula related to formula 2 that can be used when we know the final velocity instead of the initial.

\[ d = v_f t - \frac{1}{2} a t^2 \]

- Notice that the differences are final instead of initial velocity, and the minus sign instead of addition.
- Otherwise, this formula is used the same way as formula 2.

Example 4: Now the F-22 is coming in for a landing. If the runway is 2500 m long, and the plane uses the whole length to come to a stop in 43s, determine the acceleration of the plane.

Big thing to note here is that the plane coming to a stop means that the final velocity is zero.

\[ d = v_f t - \frac{1}{2} a t^2 \]
\[ d = -\frac{1}{2} a t^2 \]
\[ a = \frac{-2d}{t^2} \]
\[ a = \frac{-2(2500)}{43^2} \]
\[ a = -2.7041644132 = -2.7 \text{ m/s}^2 \]
Notice the negative sign, indicating that the brakes are slowing down the plane.

**Example 5**: A car drives 83m while accelerating at 2.4m/s² for 4.9s. Determine the final velocity of the car.

We're going to have to manipulate the formula to solve for vₖ. Keep in mind that you may have to manipulate any of the formulas we are looking at.

\[
d = v_i t - \frac{1}{2} a t^2 \\
v_f = \frac{d + 0.5 a t^2}{t} \\
v_f = \frac{83 + (0.5)(2.4)(4.9)^2}{4.9} \\
v_f = 22.81878 = 23 \text{ m/s}
\]

**Formula 4**

Another very useful formula is the following...

\[v_f^2 = v_i^2 + 2ad\]

- Very handy when you have a question with both velocities, acceleration, and displacement.
- Don’t forget to do the square root at the very end if you are solving for a velocity, as the following example shows…

**Example 6**: Determine the final velocity of a car that starts at 22 m/s and accelerates at 3.78 m/s² for a distance of 45 m.

\[
v_f^2 = v_i^2 + 2ad \\
v_f^2 = 22^2 + 2(3.78)(45) \\
v_f^2 = 824.2 \\
v_f = 28.70888 = 29 \text{ m/s}
\]

Many people forget that in the last step you need to square root in order to get the velocity instead of the velocity squared!

**How to Choose the Right Formula!**

So, how do you figure out which formula to use for a particular problem?

- As you look back through the formulas, you'll see that of the five basic things we measure about the motion of an object (vᵢ, vₖ, a, t, and d), each formula only has four.
  - To choose the correct formula, figure out the one thing that you are not given and not asked for in the question. Choose the one formula that does not have that variable.
- The following table may help.
For example, let's say I had a question where I am given acceleration, displacement, and time, and asked to find initial velocity.

- The only thing I wasn't given, and I wasn't asked for, is final velocity.
- The only formula that does not have final velocity is $d = v_i t + \frac{1}{2} at^2$. This is the formula I should use.

Remember that for all of these formulas, you may be required to manipulate the formula to find the answer you are looking for.

- Always follow the rule of finding the formula that has all the knowns and unknown that you have.
- Write down the original formula as it appears on the data sheet.
- Then manipulate it for your unknown, and solve.

**Example 7:** A paint can is knocked off the top of a building. If it falls for 3.5 s, determine the height of the building.

$$d = v_i t + \frac{1}{2} at^2$$

$$d = 0 + 0.5(-9.81)(3.5^2)$$
$$d = -60.08625 = -60 \text{ m}$$

What we should do now is “reword” the answer to say that since the paint fell 60 m (that’s why the answer is negative), the height of the building must be 60 m (just stated as a positive answer).

**Homework**

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