

Lesson 11: The Other Formulas

There are several other formulas that are very useful when the acceleration is uniform.

- Do not use these equations if the acceleration is changing! The acceleration must be constant.
- These other formulas are based on combinations of the basic velocity and acceleration formulas, as well as interpreting graphs.
- Although the formulas are shown here in a particular numbered order, you do *not* need to identify them this way.

Formula 1

The first formula is based on knowing information about displacement, final and initial velocities, and time.

$$d = \left(\frac{v_f + v_i}{2} \right) t$$

In your text book, this formula is written slightly differently as $d = 1/2 (v_i + v_f) t$. It is still the exact same formula.

- At first this may seem to be an odd acceleration formula, since acceleration does not appear in the formula as a variable.
 - Notice that there are two velocities, v_f and v_i , so we know that there must be acceleration.
 - Only use this formula when you know for certain that the object has been going through a constant acceleration, even though “a” doesn’t appear in the formula.

Example 1: Determine how far a vehicle moved if it started at 12 m/s and accelerated up to 28 m/s in a time of 34s.

$$d = \left(\frac{v_f + v_i}{2} \right) t$$
$$d = \left(\frac{28 + 12}{2} \right) 34$$
$$d = 680 = 6.8 \times 10^2 \text{ m}$$

Formula 2

We will do problems where we have information about displacement, initial velocity, time, and acceleration. The formula for these situations is...

$$d = v_i t + \frac{1}{2} a t^2$$

- Be careful with this formula. Only the time is squared in the last term, not acceleration and time.
- As a bonus, a lot of the time v_i will be zero, which cancels out the first term and leaves you with...

$$d = \frac{1}{2} a t^2$$

Example 2: Occasionally the US Air Force calls me in to test fly their “birds”. A few weeks back I was flying along in my F-22 at 97m/s when I decide to kick in the afterburners for 12.3s. If the afterburners can generate enough thrust to accelerate the F-22 at 26m/s², **determine** how far I travelled during that time.



$$\begin{aligned} d &= v_i t + \frac{1}{2} a t^2 \\ &= (97) (12.3) + (0.5) (26) (12.3)^2 \\ d &= 3159.87 = 3.2e3 \text{ m} \end{aligned}$$

Example 3: I am in an F-22 that is on the runway. From rest, I accelerate the plane at 3.9m/s². After the plane has travelled 410 m along the runway I reach take off velocity and leave the ground. **Determine** how much time my takeoff from the runway took.

This is an example of a question where the initial velocity is zero (since I’m starting from rest), so...

$$\begin{aligned} d &= v_i t + \frac{1}{2} a t^2 \\ d &= \frac{1}{2} a t^2 \\ t &= \sqrt{\frac{2d}{a}} \\ t &= \sqrt{\frac{2(410)}{3.9}} \\ t &= 14.500221 = 15 \text{ s} \end{aligned}$$

Formula 3

There is a formula related to formula 2 that can be used when we know the final velocity instead of the initial.

$$d = v_f t - \frac{1}{2} a t^2$$

- Notice that the differences are final instead of initial velocity, and the minus sign instead of addition.
- Otherwise, this formula is used the same way as formula 2.

Example 4: Now the F-22 is coming in for a landing. If the runway is 2500 m long, and the plane uses the whole length to come to a stop in 43s, **determine** the acceleration of the plane.

Big thing to note here is that the plane coming to a stop means that the final velocity is zero.

$$\begin{aligned}d &= v_f t - \frac{1}{2} a t^2 \\d &= -\frac{1}{2} a t^2 \\a &= \frac{-2d}{t^2} \\a &= \frac{-2(2500)}{43^2} \\a &= -2.7041644132 = -2.7 \text{ m/s}^2\end{aligned}$$

Notice the negative sign, indicating that the brakes are slowing down the plane.

Example 5: A car drives 83m while accelerating at 2.4m/s^2 for 4.9s. Determine the final velocity of the car.

We're going to have to manipulate the formula to solve for v_f . Keep in mind that you may have to manipulate any of the formulas we are looking at.

$$\begin{aligned}d &= v_f t - \frac{1}{2} a t^2 \\v_f &= \frac{d + 0.5 a t^2}{t} \\v_f &= \frac{83 + (0.5)(2.4)(4.9)^2}{4.9} \\v_f &= 22.81878 = 23 \text{ m/s}\end{aligned}$$

Formula 4

Another very useful formula is the following...

$$v_f^2 = v_i^2 + 2ad$$

- Very handy when you have a question with both velocities, acceleration, and displacement.
- Don't forget to do the square root at the very end if you are solving for a velocity, as the following example shows...

Example 6: **Determine** the final velocity of a car that starts at 22 m/s and accelerates at 3.78 m/s^2 for a distance of 45 m.

$$\begin{aligned}v_f^2 &= v_i^2 + 2ad \\v_f^2 &= 22^2 + 2(3.78)(45) \\v_f^2 &= 824.2 \\v_f &= 28.70888 = 29 \text{ m/s}\end{aligned}$$

Many people forget that in the last step you need to square root in order to get the velocity instead of the velocity *squared*!

How to Choose the Right Formula!

So, how do you figure out which formula to use for a particular problem?

- As you look back through the formulas, you'll see that of the five basic things we measure about the motion of an object (v_f , v_i , a , t , and d), each formula only has four.
 - To choose the correct formula, figure out the one thing that you are *not* given and *not* asked for in the question. Choose the one formula that does *not* have that variable.
- The following table may help.

Formula	a	v_f	v_i	d	t
$a = \frac{v_f - v_i}{t}$	*	*	*	X	*
$d = \left(\frac{v_f + v_i}{2}\right)t$	X	*	*	*	*
$d = v_i t + \frac{1}{2}at^2$	*	X	*	*	*
$d = v_f t - \frac{1}{2}at^2$	*	*	X	*	*
$v_f^2 = v_i^2 + 2ad$	*	*	*	*	X

For example, let's say I had a question where I am given acceleration, displacement, and time, and asked to find initial velocity.

- The only thing I wasn't given, and I wasn't asked for, is final velocity.
- The only formula that does not have final velocity is $d = v_i t + \frac{1}{2}at^2$. This is the formula to use.

Remember that for all of these formulas, you may be required to manipulate the formula to find the answer you are looking for.

- Always follow the rule of finding the formula that has all the knowns and unknown that you have.
- Write down the original formula as it appears on the data sheet.
- Then manipulate it for your unknown, and solve.

There can be *special situations* where, *in two separate parts*, we need to use uniform and accelerated motion formulas...

Example 7: You are riding in a bus that is moving at 50 km/h [forwards]. The light turns red up ahead. The bus driver takes 1.8 s to react and move their foot to the brake pedal. When they press on the brakes, the bus slows to a stop with an acceleration of 1.93 m/s^2 [backwards]. **Determine** the total distance the bus travelled.

Uniform Motion (Before Braking)

$$v = \frac{d}{t}$$

$$d = vt$$

$$d = 13.8888(1.8)$$

$$d = 25 \text{ m}$$

Accelerated motion (While Braking)

$$v_f^2 = v_i^2 + 2ad$$

$$d = \frac{v_f^2 - v_i^2}{2a}$$

$$d = \frac{0^2 - 13.8888^2}{2(-1.93)}$$

$$d = 49.9744 \text{ m}$$

Total distance travelled = $25 + 49.9744 = 74.9744 = \mathbf{75 \text{ m [forwards]}}$

Homework

p.48 #1,2
p.50 #1,2

p.51 #1,2
p.52 #1,2

p.53 all odd