

Lesson 11: The Other Formulas

There are several other formulas that are very useful when the acceleration is uniform.

- Do not use these equations if the acceleration is changing! The acceleration must be constant.
- These other formulas are based on combinations of the basic velocity and acceleration formulas, as well as interpreting graphs.
- Although the formulas are shown here in a particular numbered order, you do *not* need to identify them this way.

Formula 1

The first formula is based on knowing information about displacement, final and initial velocities, and time.

$$d = \left(\frac{v_f + v_i}{2} \right) t$$

In your text book, this formula is written slightly differently as $d = 1/2 (v_f + v_i) t$. It is still the exact same formula.

- At first this may seem to be an odd acceleration formula, since acceleration does not appear in the formula as a variable.
 - Notice that there are two velocities, v_f and v_i , so we know that there must be acceleration.
 - Only use this formula when you know for certain that the object has been going through a constant acceleration, even though “a” doesn't appear in the formula.

Example 1: Determine how far a vehicle moved if it started at 12 m/s and accelerated up to 47 m/s in a time of 34s.

$$d = \left(\frac{v_f + v_i}{2} \right) t$$
$$d = \left(\frac{47 + 12}{2} \right) 34$$
$$d = 1.0e3 \text{ m}$$

Formula 2

We will do problems where we have information about displacement, initial velocity, time, and acceleration. The formula for these situations is...

$$d = v_i t + \frac{1}{2} a t^2$$

- Be careful with this formula. Only the time is squared in the last term, not acceleration and time.
- As a bonus, a lot of the time v_i will be zero, which cancels out the first term and leaves you with...

$$d = \frac{1}{2} a t^2$$

Example 2: Occasionally the US Air Force calls me in to test fly their “birds”. A few weeks back I was flying along in my F-22 at 97m/s when I decide to kick in the afterburners for 12.3s. If the afterburners can generate enough thrust to accelerate the F-22 at 26m/s², **determine** how far I traveled during that time.



$$\begin{aligned}d &= v_i t + \frac{1}{2} a t^2 \\ &= (97\text{m/s})(12.3\text{s}) + \frac{1}{2} (26\text{m/s}^2)(12.3\text{s})^2 \\ d &= 3.2\text{e}3 \text{ m}\end{aligned}$$

Example 3: I am in a F-22 that is on the runway. From rest, I accelerate the plane at 3.9m/s² for 9.5s to reach take off velocity. **Determine** how long the runway must be.

This is an example of a question where the initial velocity is zero (since I’m starting from rest), so...

$$\begin{aligned}d &= v_i t + \frac{1}{2} a t^2 \\ d &= \frac{1}{2} a t^2 \\ &= \frac{1}{2} (3.9\text{m/s}^2)(9.5\text{s})^2 \\ d &= 1.8\text{e}2 \text{ m}\end{aligned}$$

Formula 3

There is a formula related to formula 2 that can be used when we know the final velocity instead of the initial.

$$d = v_f t - \frac{1}{2} a t^2$$

- Notice that the differences are final instead of initial velocity, and the minus sign instead of addition.
- Otherwise, this formula is used the same way as formula 2.

Example 4: A car drives 83m while accelerating at 2.4m/s² for 4.9s. Determine the final velocity of the car.

We're going to have to manipulate the formula to solve for v_f . Keep in mind that you may have to manipulate any of the formulas we are looking at.

$$\begin{aligned}d &= v_f t - \frac{1}{2} a t^2 \\ v_f &= \frac{d + 0.5 a t^2}{t} \\ v_f &= \frac{83 + (0.5)(2.4)(4.9)^2}{4.9} \\ v_f &= 23 \text{ m/s}\end{aligned}$$

Formula 4

Another very useful formula is the following...

$$v_f^2 = v_i^2 + 2ad$$

- Very handy when you have a question with both velocities, acceleration, and displacement.
- Don't forget to do the square root at the very end if you are solving for a velocity, as the following example shows...

Example 5: Determine the final velocity of a car that starts at 42 m/s and accelerates at 3.78 m/s² for a distance of 12 m.

$$\begin{aligned} v_f^2 &= v_i^2 + 2ad \\ v_f^2 &= 42^2 + 2(3.78)(12) \\ v_f^2 &= 1855 \\ v_f &= 43\text{m/s} \end{aligned}$$

Many people leave the answer like this, forgetting that this is velocity *squared*!

How to Choose the Right Formula!

So, how do you figure out which formula to use for a particular problem?

- As you look back through the formulas, you'll see that of the five basic things we measure about the motion of an object (v_f , v_i , a , t , and d), each formula only has four.
 - To choose the correct formula, figure out the one thing that you are **not** given and **not** asked for in the question. Choose the one formula that does **not** have that variable.
- The following table may help.

Formula	a	v_f	v_i	d	t
$a = \frac{v_f - v_i}{t}$	*	*	*	X	*
$d = \left(\frac{v_f + v_i}{2} \right) t$	X	*	*	*	*
$d = v_i t + \frac{1}{2} at^2$	*	X	*	*	*
$d = v_f t - \frac{1}{2} at^2$	*	*	X	*	*
$v_f^2 = v_i^2 + 2ad$	*	*	*	*	X

For example, let's say I had a question where I am given acceleration, displacement, and time, and asked to find initial velocity.

- The only thing I wasn't given, and I wasn't asked for, is final velocity.
- The only formula that does not have final velocity is $d = v_i t + \frac{1}{2} at^2$. This is the formula I should

use.

Remember that for all of these formulas, you may be required to manipulate the formula to find the answer you are looking for.

- Always follow the rule of finding the formula that has all the knowns and unknown that you have.
- Write down the original formula as it appears on the data sheet.
- Then manipulate it for your unknown, and solve.

Example 6: Determine the displacement of a car that starts at 10 m/s and accelerates at 1.89m/s² and reaches a final speed of 32m/s.

$$v_f^2 = v_i^2 + 2ad$$
$$d = \frac{(v_f^2 - v_i^2)}{2a}$$
$$d = \frac{(32^2 - 10^2)}{2(1.89)}$$
$$d = 2.4e2 m$$

Can you see why I am using this formula?