

Lesson 24: Newton's Second Law (Motion)

To really appreciate Newton's Laws, it sometimes helps to see how they build on each other.

- The First Law describes what will happen if there is *no force*.
- The Second Law describes what will happen if there *is a force*.

The Second Law (The Law of Motion)

“When an external, unbalanced force acts on an object, the object will **accelerate in the same direction as the force**. **The acceleration varies directly as the force**, and **inversely as the mass**.”

“When an external, unbalanced force...”

We are still talking about these external forces, but now we've added in the idea of “unbalanced”.

- Unbalanced just means that there isn't anyone (or anything) pushing against the force hard enough to cancel it out.

“...accelerates in the same direction as the force...”

Some people think this means that the object will move in the same direction as the force... not necessarily!

- The object might be moving to the right, while a force is pushing left.
- That means the object will slow down.
- It's acceleration is in the direction of the force (to the left), but it is still moving to the right.

“The acceleration varies directly as the force...”

This just means that if the force increases, the acceleration will increase. If the force decreases, the acceleration decreases.

- This makes sense... push something harder and it will accelerate more! They depend directly on each other.

“...and inversely as the mass.”

This means that if the mass is bigger, the acceleration is less. If the mass is less, the acceleration is more.

- This makes sense also... if something has less mass, it is easier to make it move faster! They depend inversely on each other.

Mathematically this would be written as...

$$a \propto F$$
$$a \propto \frac{1}{m}$$

- The little swimming fish symbol (\propto) is the Greek letter “alpha” and means “varies as” in math.
- It does *not* mean they are equal to each other, but it does show they are related to each other in some way.

- We are showing that acceleration is directly related to force
 - If the net force exerted is increased, the acceleration will also increase.
- Acceleration is inversely related to mass.
 - If the mass increases, the acceleration will decrease.
 - If the mass is bigger, the object will have more inertia, so it will resist changes to its motion more.

When you combine these two relations, you get one of the most basic and important formulas ever discovered in physics.

$$F = ma$$

F = force (**Newton**, **N**)
 m = mass (**kg**)
 a = acceleration (**m/s²**)

The unit of force is called the Newton.

- It's equivalent to kg m/s².
- It was named in honor of the outstanding work that Newton did in physics.
- By definition a one kilogram mass will be accelerated at 1 m/s² if a 1 Newton force is applied to it.

Example 1: Determine the force acting on a 5.46kg object if it is accelerated at 17m/s².

$$F = ma$$

$$F = (5.46)(17)$$

$$F = 93\text{N}$$

Example 2: Determine the acceleration of a 1000 kg car if a 2.5e3 N force acts on it?

$$F = ma$$

$$a = \frac{F}{m} = \frac{2.5e3}{1000} = 2.5 \text{ m/s}^2$$

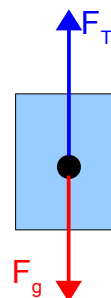
Applications of Newton's Second Law

It is best to start the more complex diagrams by analyzing what forces are involved, come up with a net force formula, and draw a free body diagram.

- In these examples the forces will all be acting vertically *or* horizontally, not both.

Example 5: A 3.70e3 kg elevator is being raised by a cable that exerts a 4.00e4 N force upwards. **Determine** the acceleration of the elevator.

- There are only two forces acting on the elevator, the force of tension in the cable pulling it up and the force of gravity acting down.
- It is the net force that causes the acceleration of the elevator, so we're basically solving for that. We can substitute in $F_{\text{NET}} = ma$
- Remember to use negative for down and positive for up.



Drawing 1: Free body diagram of elevator.

$$\begin{aligned}
 F_{NET} &= F_g + F_T \\
 ma &= mg + F_T \\
 a &= \frac{mg + F_T}{m} \\
 a &= g + \frac{F_T}{m} \\
 a &= -9.81 + \frac{4.00e4}{3.70e3} = -9.81 + 10.8 \\
 a &= 1.0 \text{ m/s}^2
 \end{aligned}$$

You don't need to do all this fancy canceling (as shown on the fourth line). You can just start substituting in your numbers.

The acceleration of the elevator is 1.0 m/s^2 [upwards].

Example 6: Determine the acceleration of the elevator from example 5 if the tension in the cable is only $3.30e4 \text{ N}$.

$$\begin{aligned}
 F_{NET} &= F_g + F_T \\
 ma &= mg + F_T \\
 a &= \frac{mg + F_T}{m} \\
 a &= g + \frac{F_T}{m} \\
 a &= -9.81 + \frac{3.30e4}{3.70e3} = -9.81 + 8.91 \\
 a &= -0.89 \text{ m/s}^2
 \end{aligned}$$

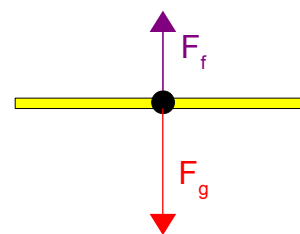
The acceleration of the elevator is now 0.89 m/s^2 [down].

Example 7: A 0.0500 g piece of paper is dropped. While it falls there is a frictional force of $4.71e-4 \text{ N}$. Determine the acceleration of the paper.

- The frictional force must be pointing upwards since it will resist the motion of the paper falling down.
- Don't forget to turn the mass into kilograms.

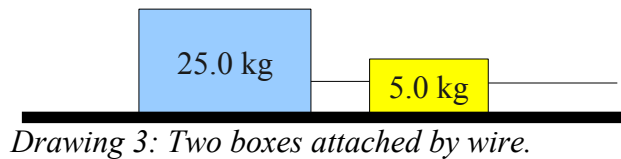
$$\begin{aligned}
 F_{NET} &= F_g + F_f \\
 ma &= mg + F_f \\
 a &= \frac{mg + F_f}{m} \\
 a &= g + \frac{F_f}{m} \\
 a &= -9.81 + \frac{4.71e-4}{5.00e-5} = -9.81 + 9.42 \\
 a &= -0.39 \text{ m/s}^2
 \end{aligned}$$

The acceleration of the paper is 0.39 m/s^2 [down].



Drawing 2: Free body of falling sheet of paper.

Example 8: Two boxes are attached by a thin wire as shown in Drawing 3. A person pulls on the wire attached to the yellow box. There is a force of friction of 213 N acting on the boxes. **Determine** the force applied to the boxes if they accelerate to the right at 2.45 m/s^2 .



Drawing 3: Two boxes attached by wire.

- Since the boxes are attached by a wire, they must accelerate at the same rate. We can also treat them as one big mass instead of two small masses.
- We will assume that the wires have no mass.
- F_g and F_N cancel each other vertically, so we can ignore them.
- To the right is positive, and to the left is negative.

$$F_{NET} = F_a + F_f$$

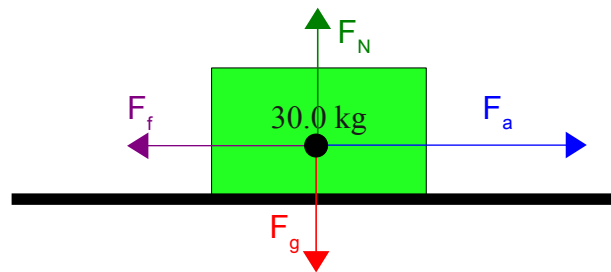
$$ma = F_a + F_f$$

$$F_a = ma - F_f$$

$$F_a = 30.0(2.45) - (-213)$$

$$F_a = 287 \text{ N}$$

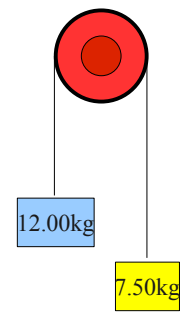
The applied force is 287 N [right].



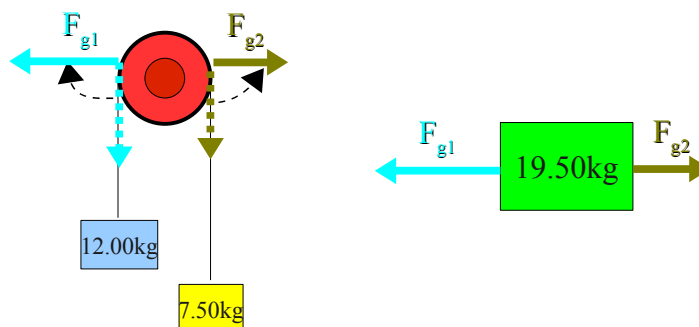
Drawing 4: Free body diagram of single mass.

Example 9: “Atwood's Pulley” is a classic physics problem involving two unequal masses hanging from a wire over a pulley. Since one of the masses is heavier, the whole thing will move so that the heavier mass moves down and the lighter mass moves up. If mass one is 12.00 kg and mass two is 7.50 kg, **determine** the acceleration of each mass.

- Since we are dealing with two masses that are attached, we will do the same thing as **Example 8** and add the masses.
- We will need to be careful, since the net force is pulling the blue box down and the yellow box up, but is made up of two forces due to gravity pulling down on different sides.
- To simplify this, we will take our single mass and pretend that one force due to gravity is pulling it to the left, while the other force due to gravity is pulling it to the right.



Drawing 5: Atwood's Pulley.



Drawing 6: Rotate the two forces due to gravity horizontally, and show them acting on the combined mass.

Mass One

$$F_{g1} = mg$$

$$F_{g1} = 12.00(-9.81)$$

$$F_{g1} = -117.72 \text{ N}$$

Pulls to the left with 117.72N.

Mass Two

$$F_{g2} = mg$$

$$F_{g2} = 7.50(9.81)$$

$$F_{g2} = 73.575 \text{ N}$$

Pulls to the right with 73.575N.

$$F_{NET} = F_{g1} + F_{g2}$$

$$F_{NET} = -117.72 + 73.575$$

$$F_{NET} = -44.145$$

$$F_{NET} = -44.15 \text{ N}$$

$$F_{NET} = ma$$

$$a = \frac{F_{NET}}{m} = \frac{-44.15}{19.50}$$

$$a = -2.264 \text{ m/s}^2$$

The minus sign, based on Drawing 6, just shows that mass one will drop as mass two rises. Both will have an acceleration of 2.264 m/s^2 .

When doing questions that involve a pulley, just remember that a pulley kind of changes the direction a force is acting in. That's why we can change the directions some of the forces are pointing in these questions, even though we typically can not change the direction of vectors.