Lesson 2: Archimedes' Principle

Archimedes of Syracuse is probably one of the most important scientists and mathematicians of all time.

- Although he lived over 2200 years ago, he was able to make some fundamental discoveries in the study of physics (specifically fluids).
- There are also a number of stories that surround these discoveries, some of which have probably been, ahem, *embellished* over the centuries.

Illustration 1: "Archimedes Thoughtful" by the artist Fetti (1620).

The Golden Crown

Supposedly King Hiero II had commissioned a goldsmith to make him a new crown in the shape of a laurel wreath.

- The king had some reason to think that the goldsmith had been dishonest and had mixed in some silver with the gold.
- Although the general idea of density was already known, they had no way of measuring the volume of the crown accurately since it was such an irregular shape. Without knowing the exact volume of the crown, it was impossible to know what the mass of the crown *should* be if it was solid gold.
- Archimedes was asked to find a way to measure the volume of the crown without damaging it in any way.
    - The solution came to him as he was taking a bath. He noticed that as he got in to the tub, the water level would rise.
    - He figured out that the amount of volume of his body submerged displaced an equal volume of water.
    - All he needed to do was submerge the crown in water, and measure the volume of water it pushed out of the way. This would be equal to the volume of the crown.
- As the story goes, he was so excited by his discovery that he jumped out of the tub and ran naked through the streets yelling "Eureka!" which translates from Greek as "I have found it."

The whole point of this is that it leads to the discovery of **Archimedes' Principle**, which says that the force of buoyancy that an object will feel when it is immersed in a fluid is equal to the weight of the of the fluid displaced by the object.

- This just means that if you place an object with a volume of (for example) 3 m^3 fully into some water, it will displace 3 m^3 of water. The buoyant force the object will feel is equal to the weight of that 3 m^3 of water.
    - It is important to note that only the volume of the object that is submerged into the fluid can displace any volume, so that is the only amount considered when you figure out the buoyant force.

We can figure out a formula for **buoyancy** based on the formula we have for **weight**.

- Keep in mind that the buoyancy depends on the mass of the *fluid* displaced.
- Just like in the last lesson, we can use $\mathbf{m = \rho V}$. This is handy, since we often talk about fluids in terms of density and volume, instead of mass.

$$F_g = mg$$
$$F_B = m_{fluid}\, g$$
$$F_B = \rho_{fluid} V_{\text{sub}}\, g$$

Example 1: A motor boat is being launched at a lake. When it is placed in the water, it sinks into the water enough to displace 4.3m^3 of water. Assuming that this is enough for the boat to float, **determine** the mass of the motor boat.

If the boat is floating, that must mean the force of gravity pulling it down is balanced by an equal magnitude of buoyant force acting upward. We will only deal with the magnitudes of the forces.

$$F_B = \rho_{water} V_{\text{sub}}\, g$$
$$F_B = 1.000\text{e}3(4.3)(9.81)$$
$$F_B = 42183\, N = 4.2\text{e}4\, N$$

This buoyant force is equal to the weight of the boat (for it to be floating), so we can use it to figure out the mass of the boat.

$$F_g = m_{boat}\, g$$
$$m = \frac{F_g}{g}$$
$$m = \frac{42183}{9.81}$$
$$m = 4300 = 4.3\text{e}3\, kg$$