6: Angular Momentum

We can come up with a formula for calculating angular momentum based on the formula we have for translational momentum.

Translational	**Rotational**
$p = mv$	$L = I\omega$

p = translational momentum (kg m/s)
m = mass (kg)
v = velocity (m/s)
L = angular momentum (kg m^2 rad/s)
I = moment of inertia (kg m^2)
ω = angular velocity (rad/s)

Example 1: A particular car tire has a moment of inertia of 0.60 kg m^2. If the tire is spinning at 5.5 Hz, **determine** the angular momentum of the tire.

First we will find the angular velocity…

$$\omega = 2\pi f = 2(3.14)(5.5) = 34.54\, rad/s$$

Then we calculate the angular momentum…

$$L = I\omega = 0.60(34.54) = 20.724 = 21\, kg\, m^2\, rad/s$$

Previously, you have looked at **translational conservation of momentum** examples involving 1D and 2D **collisions** of two or more objects.

- In the **rotational** examples we will look at here, **we do *not* have collisions**. Instead, we examine the **conservation of angular momentum** for a single object as it spins.
- The conservation of angular momentum is stated in a way that is very similar to the way we have looked at conservation laws before.

The total angular momentum of a rotating body will remain constant if the net torque acting on the body is zero.

- The first line just tells us that if we measure the angular momentum of the object initially we should find that it is the same later.
- The second line points out this is only true if there is no external force (torque) acting on it to change its rotation.

The best example you have probably seen of this is a figure skater doing a spin.

- *Please note, the real numbers involved in these calculations for a figure skater get a lot more complicated, because a figure skater is not a simple object that we can easily calculate values like moment of inertia for. We are using some simplified ideas here that help us appreciate what is happening from the standpoint of conservation of angular momentum.*

When a figure skater is doing a spin with their **arms stretched out**, they will be rotating slower.

- Stretching out their arms makes their “radius” bigger, so they have a big moment of inertia.
- Their angular velocity is small (they are not spinning too quickly).

$$L = \text{\Large I}\, \text{\tiny ω}$$

A big moment of inertia with a small angular velocity…

When the figure skater wants to spin faster, they **pull their arms in close** to themselves.

- Pulling their arms in close to their body makes their “radius” smaller, so they have a smaller moment of inertia.
- In order to conserve their angular momentum, their angular velocity makes up for this by getting bigger (they speed up)!

$$L = \text{\tiny I}\, \text{\Large ω}$$

…or a small moment of inertia with a big angular velocity.

This means that the figure skater has conserved their angular momentum.

- Although there is some torque from friction with the ice on their skates and their body with the air, this is negligible and can be ignored.
- As we did for previous conservation laws, we can write out that what we started with equals what we end with. Remember, in AP physics “L_o” means initial angular momentum.

$$L_o = L$$
$$I_o \omega_o = I \omega$$

Example 2: A figure skater is practicing their spins. With their arms stretched out they have a moment of inertia of 0.80 kg m^2 and is spinning at 15 rad/s. If they pull in their arms so they have a moment of inertia of 0.25 kg m^2, **determine** their new angular velocity.

$$L_o = L$$
$$I_o \omega_o = I \omega$$
$$\omega = \frac{I_o \omega_o}{I}$$
$$\omega = \frac{0.80(15)}{0.25}$$
$$\omega = 48\, rad/s$$