# Lesson 1: Momentum & Impulse

How does a karate expert chop through cement blocks with a bare hand?
Why does a fall onto a trampoline hurt less than onto a cement floor?
Why do people in larger vehicles usually end up with fewer injuries in accidents?

It's easy to come up with answers like…

"The karate guy is strong!"
"Trampolines are softer!"
"Bigger is better!"

...but have you ever stopped to consider the why? That's when physics comes walking in, waving explanations in everyone's face.

Spend a couple minutes right now to come up with explanations of the three situations using physics principles you have learned so far. Keep these situations and your explanations in mind as you cover this section on momentum and impulse. See if you need to modify or change your explanations based on what you learn.

## Momentum

Momentum is an idea that combines mass and velocity into one package. It is an idea that is similar to **inertia** and **kinetic energy**.

> **Inertia** is the property of an object to stay at rest or in motion.
>
> **Kinetic energy** is the amount of energy that an object has due to its motion. ($E_k = \frac{1}{2} mv^2$)

Momentum is not truly either of these, but ends up like a mix of the two.

- If you compare and contrast momentum and **kinetic energy**, you'll notice a couple things…
  - First, they both have mass and velocity in their formulas.
  - Second, **kinetic energy** has to do with ability to do work, momentum doesn't.
  - Although they are similar, they are not the same.
- We haven't given you any way to calculate inertia yet, so is momentum the same as inertia?
  - Not really. **Inertia** is a concept, not something that is directly measured.

Momentum is calculated by multiplying the mass and velocity of an object.

$$\mathbf{p = m\,v}$$

p = momentum (kg m/s)
m = mass (kg)
v = velocity (m/s)

- Notice that momentum does not have a nice derived unit, although I would appreciate it if you lobbied physicists to name it the "Clintberg" in my honor. You'll just need to use the units "kg m/s" until we can change this ;)

**Example 1**: A 1000 kg car is moving at 10km/h. **Determine** the momentum of the car.

p = mv
p = 1000 (2.777777)
p = 2777.77777 = 2.8e3 kg m/s

**Example 2**: A meteor moving through the Earth's atmosphere has a momentum of 1.15e4 kgm/s. As it falls, friction with the atmosphere slows it to 1/4 its original speed, as its mass shrinks to 2/9 of its original mass. **Determine** its new momentum.

To solve questions like this, always think of the classic "if I do it to one side, I have to do it to the other" in math. We are doing some stuff to the variables on the right side, so lets do the same to the left...

$$p = m v$$
$$\left(\frac{1}{4}\right)\left(\frac{2}{9}\right)p = \left(\frac{1}{4}\right)m\left(\frac{2}{9}\right)v$$
$$\frac{2}{36}p = \frac{2}{36}mv$$
$$\frac{1}{18}p = \frac{1}{18}mv$$

All together, we have really just multiplied both sides by 1/18. Take the original amount of momentum given in the question and divide it by 18...

$$\frac{1}{18}(1.15\text{e}4) = 639\ kgm/s$$

Note that we have kept the sig digs based on the original value that was given.

## Impulse

The simple definition for **impulse** is that it is a **change in momentum**.

- Since impulse means the momentum has changed, the object must be moving at a different velocity (like in the last example)
  - We will assume that the mass of the object usually stays the same.
- As a formula this means we would change the formula for momentum to the one shown here...

$$\Delta p = m\,\Delta v$$

Δp = impulse (*change in momentum*) (kgm/s)
m = mass (kg)
Δv = change in velocity ($v_f - v_i$) (m/s)

**Example 3**: A box of tic tacs (15g) is sliding along the table at 5.0m/s. I try to stop it, but only slow it down to 1.6 m/s. **Determine** the impulse I impart to the box.

$\Delta p = m\,\Delta v$
$\Delta p = m\,(v_f - v_i)$
$= 0.015\,(1.6s - 5.0)$
$\Delta p = -0.051$ kg m/s

The negative answer just identifies that my impulse was acting in the negative direction. Momentum was **taken away** from the object.

But wait a second, if an impulse changes the velocity of the object, that means it's ***accelerating***.

- Acceleration of an object can only occur if a force is acting on the object… so force must be related to impulse in some way.
- This leads us to the link to Newton.

## Newton's Second Law

When Newton came up with his Second Law of Motion, he didn't write it in the form we usually see it today, F = ma.

- Remember that he was playing around with some new ideas, and didn't necessarily look for the "easiest" way to state his theories.
- Instead, he kept talking about the "*quantity of motion*" of an object, what we today call momentum.
- When he stated his Second Law he said the force is proportional to the rate of change in the momentum.

$$F = \frac{\Delta p}{\Delta t}$$

Notice that you can solve this formula to get what we now consider the "standard" form of the Second Law…

$$F = \frac{\Delta p}{\Delta t} \quad \text{but we know that} \quad \Delta p = m\,\Delta v$$

$$F = \frac{m \Delta v}{\Delta t} \quad \text{and we also know that} \quad a = \frac{\Delta v}{\Delta t}$$

$$F = ma$$

We can also come up with a different (and more versatile) version of the impulse formula.

- The formula you were first given was…

$$\Delta p = m\,\Delta v$$

- But we just saw that Newton used impulse in his formulas…

$$F = \frac{\Delta p}{\Delta t}$$

which becomes

$$\Delta p = F\,\Delta t$$

We can stick these two formulas together to get the formula as it is shown on your data sheet (in the *Dynamics* section).

$$\mathbf{F\,\Delta t = m\,\Delta v}$$

The great thing about this formula is that you can basically use it three ways.

- Use the whole formula if you have three of the variables and are looking for the fourth.
- Use just the left hand side as an impulse formula.
- Use just the right hand side as an impulse formula.

**Example 4**:A rifle is firing a 9.00 g bullet so that it leaves the muzzle after 3.00e-2 s traveling at at 200 m/s. **Determine** the average force of the rifle acting on the bullet.

$$F\Delta t = m\Delta v$$

$$F = \frac{m\Delta v}{\Delta t}$$

$$F = \frac{0.00900(200-0)}{3.00\text{e-}2}$$

$$F = 60.0\,N$$

AP Students...
Look at how you could solve this without using a calculator...

You can see that a change in momentum (impulse) depends on two factors… force and time interval.

- To change an object's momentum, think of the following situations:

1. You could apply a medium force over a medium time interval.

$$F\ \Delta t = \Delta p$$

2. You could apply a big force over a small time interval and get the same impulse as in (1).

$$\text{\Huge F}\ {\scriptstyle \Delta t} = \Delta p$$

3. Or, you could apply a small force over a long time interval and still get the same impulse.

$${\scriptstyle F}\ \text{\Huge $\Delta$t} = \Delta p$$

This explains why you would want to come to a stop by hitting a haystack instead of a brick wall with your car.

- In each case the impulse is the same (your mass stays the same, your $\Delta v$ stays the same).
- When you hit the brick wall…

$$\text{\Huge F}\ {\scriptstyle \Delta t} = \Delta p$$

- Youch! All that force on your body is going to hurt! The impulse happened in a very short time period.

- When you hit the haystack…

$${\scriptstyle F}\ \text{\Huge $\Delta$t} = \Delta p$$

- Not much force at all, since the impulse is spread out over a long time period!

It's the force that "hurts", so you want it to be as small as possible.

- You can use the same argument to explain hitting an airbag instead of a steering wheel, using a bungee cord instead of a rope, or falling onto a wooden floor instead of a cement one.

**Example 5**: A 75kg man is involved in a car accident. He was initially traveling at 65km/h when he hit a large truck.

Whether or not he has an airbag, it will take the same amount of impulse to stop the person during the collision.

$$\Delta p = m \Delta v$$
$$\Delta p = 75(0 - 18.05555) = -1354.16\, kgm/s$$

a) If he had no airbag in his car and he came to rest against the steering wheel in 0.050s, **determine** how much force was exerted on his body.

First, change 65 km/h into 18.05555 m/s.

$$F \Delta t = m \Delta v$$
$$\Delta p = F \Delta t$$
$$F = \frac{\Delta p}{\Delta t}$$
$$F = \frac{-1354.16}{0.050}$$
$$F = -27083.33333 = -2.7e4\, N$$

b) If he did have an airbag that inflated and deflated correctly, bringing him to rest over a time of 0.60s, **determine** how much force was exerted on his body.

$$F \Delta t = m \Delta v$$
$$\Delta p = F \Delta t$$
$$F = \frac{\Delta p}{\Delta t}$$
$$F = \frac{-1354.16}{0.60}$$
$$F = -2256.944444 = -2.3e3\, N$$

Which is less than 10% of the force felt without an airbag… a definite improvement!

## *Homework*

p451 #1, 2

p452 #1*

p453 #2, 3, 5, 8, 11

p458 #1