Lesson 2: Impulse Graphs

So far we've implied some things about what is constant and what can change in the impulse formula \( F \Delta t = m \Delta v. \)

- We look at situations where we expect the mass of the object will stay constant.
- The velocity will change, and that's why we put a delta in front of it.
- Time is changing (sort of) as we measure it over a period of time.
- Force must be a constant. We assume that the force being exerted on the object was always the same, causing a constant acceleration.

If we are looking at a simple impulse question (where the force is constant), we can figure out exactly what we can interpret from a graph.

- Later this may help us to figure out a more complicated question, like if the force changes.

The following graph is an example of one of those simple situations where the force remains constant during the entire time.

- If we look at what the slope might represent, we get...

\[
slope = \frac{rise}{run} = \frac{F}{\Delta t}
\]

- Since nothing in the impulse formula can be rearranged to give us force over time, the slope doesn't mean anything to us in this situation.

- If we look at the area under the line, we get something a bit better...

\[
Area = lw = F \Delta t = \Delta p
\]

- Since the area under the line is really just multiplying force and time, the area must be equal to the impulse acting on the object.

**Example 1**: For the graph shown above, assume that it shows a constant force of 25 N acting over a 10 s period of time. **Determine** the impulse.

Since area under the line is equal to impulse...

\[
\begin{align*}
Area &= lw \\
Area &= 25(10) \\
Area &= 2.5e2 \\
\Delta p &= 2.5e2 N \cdot s
\end{align*}
\]
If we really wanted to, we could have simply use $\Delta p = F\Delta t$ to figure out the impulse. We could do this in this situation because **the force is constant**.

- If we need to do a question where **the force is not constant**, we can still use the area under the line to get the impulse, even though the formula $\Delta p = F\Delta t$ can not be used.

**Example 2**: I am in a car that is accelerating from rest at a red light. I want to calculate the impulse that is acting on the car during the first 5.78s. If I know that the force on the car steadily increases from 0 N to 3012 N over this time, **determine** the impulse. If the mass of the car is 1500 kg, also **determine** the final velocity of the car.

Let’s start by graphing the information we were given. We will get a nice linear graph, since it said that the force **steadily** increases.

If we calculate the area under the graph (a triangle) we will know what the impulse is.

$$A = \frac{1}{2} bh$$

$$= \frac{1}{2} (5.78 \text{ s})(3012 \text{ N})$$

$$= 8704.68$$

$$A = 8.70e3 \text{ kgm/s}$$

**Illustration 2**: Graph for Example 2 (Force as a function of Time)

To calculate the final velocity, we can use the value for the impulse we just got with the right hand side of the impulse formula. Remember that the initial velocity (sitting at the light) is zero...

\[
\begin{align*}
\Delta p &= m\Delta v \\
\Delta p &= m(v_f-v_i) \\
\Delta p &= m v_f \\
v_f &= \frac{\Delta p}{m} \\
v_f &= \frac{8704.68}{1500} \\
v_f &= 5.80312 \\
v_f &= 5.80 \text{ m/s}
\end{align*}
\]

The graph that we make does not have to be a pretty right angle triangle either. We can also do some crazy stuff with what we are looking for in the question, as the next example shows.
Example 3: This graph shows the result of applying 500 kgm/s of impulse to an object as it moved across the floor for 10.0 s. Determine the maximum force that was exerted.

Even though it is not a right angle triangle, this graph still shows a triangle that we can use the regular area formula with. In this case, we already know the area (the impulse is 500 kgm/s) and we know the base (10.0 s). All we want is the height of the triangle, since that is the magnitude of the maximum force.

\[
\text{Area} = \frac{bh}{2} \\
\Delta p = \frac{F \Delta t}{2} \\
F = \frac{2 \Delta p}{t} = \frac{2(500)}{10.0} = 100 \text{ N}
\]

Even if the graph is a curved line, you can still at least estimate the area under the graph.

- Although this will only be an approximate area, without getting into calculus it's as good as you'll get and as good as you need.
  - On the graph shown below we have an s-curve that would be difficult to calculate the exact area of.
  - Instead, we just look at the triangle drawn in red. For the little bit extra it has near the beginning, it misses a bit later on. These two parts should more or less make up for each other, so that the area of the triangle will be about the same as the area under the curve.

Illustration 3: Pushing object across the floor.

Illustration 4: Instead of trying to figure out the area of the curve exactly, we just use the area of the triangle as an approximation.

**Homework**

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