Lesson 4: 2-D Collisions

We now need to turn our attention towards questions involving objects that collide in two dimensions (2D).

- In the previous section we were looking at only linear collisions (1D), which were quite a bit simpler (mathematically) to handle.
- Now we need to figure out some ways to handle calculations in more than 1D.
  - You actually learned about this in Physics 20 in the vectors section.

First, let's look at drawing some diagrams of some common collisions, then we'll worry about the calculations…

Example 1: **Sketch** a diagram that represents the collision between two moving pool balls (of equal mass) that strike each other with an angle of 30° between them. They do not stick together.

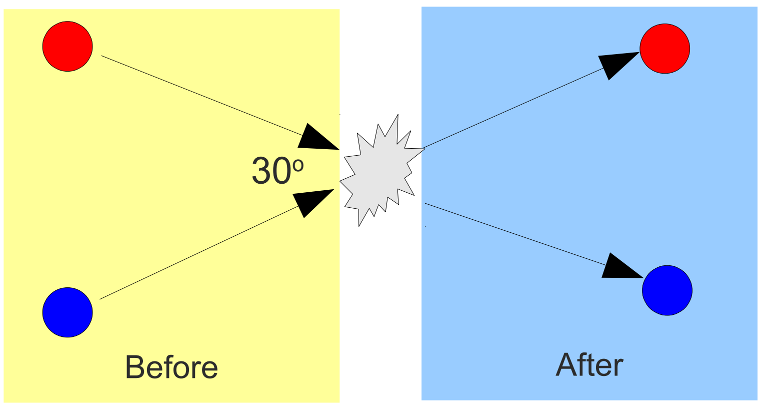

Illustration 1: Collision between two moving balls

We need to show what happens before and after the collision.

- So far this is just a rough sketch, since we weren't told anything about their velocities or the angle that they traveled away at.
- All that we're doing at this point is showing that we know that the balls should move off in directions similar to the ones shown here.

Example 2: **Sketch** a diagram that represents the collision between a moving pool ball that strikes a stationary pool ball. They move off with an angle of 60° between them.

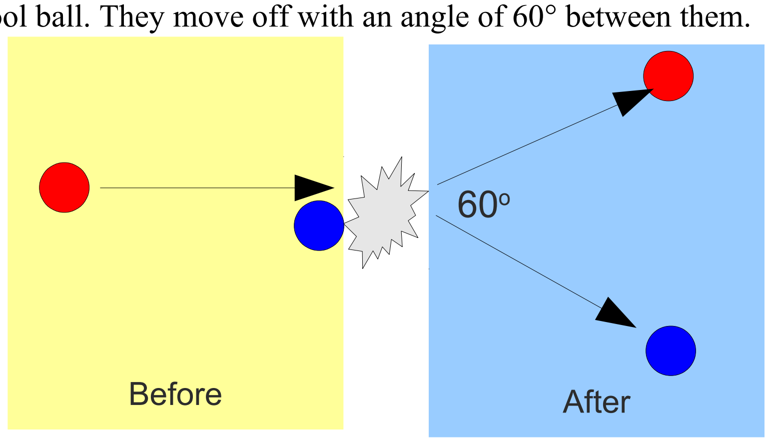

Illustration 2: Collision between one moving and one stationary ball

In examples like this you will often see that the question refers to the collision as "**glancing**."

- This simply means that the objects do ***not*** hit head on. If they did hit head on then the collision would be 1D, not 2D.

Now we need to start looking at some questions with calculations.

- Some people are more comfortable doing these questions using components, others like using cosine and sine laws.
- Since not everyone knows cosine and sine laws, we'll only use components in the following examples.

Example 3: A 1.20 kg red ball moving to the right at 17.1m/s strikes a stationary 2.31 kg blue ball. If the final velocity of the red ball is 13.5m/s at 23.0° above the horizontal, **determine** the final velocity of the blue ball.

A sketch is always a good idea, even if you're not asked for one...

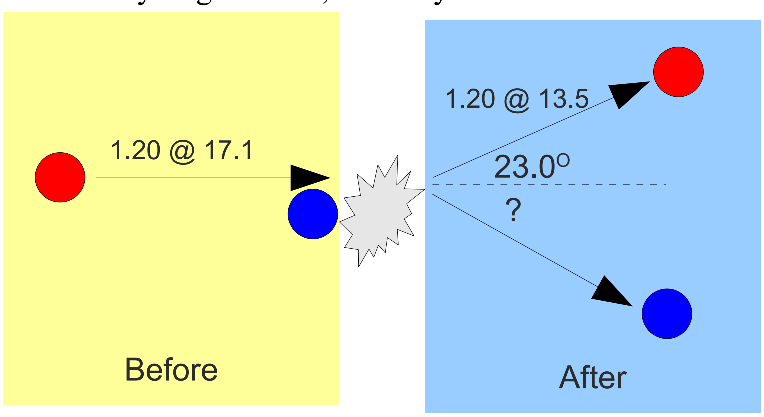

Illustration 3: Sketch of collision for example 3

The **total** momentum before has to equal the **total** momentum after. So, we need to calculate all the momentum before the collision as vectors and make it equal to the momentum after the collision.

First, we'll just figure out the momentum of the balls on the paths they're on. You **MUST** do this; you can not do anything with just the velocities.

Before	After
$\mathbf{p_r} = m_r v_r = (1.20)(17.1) = \mathbf{20.52\ kgm/s}$	$\mathbf{p_r'} = m_r v_r = (1.20)(13.5) = \mathbf{16.2\ kgm/s}$
$\mathbf{p_b} = m_b v_b = (2.31\text{kg})(0\text{m/s}) = \mathbf{0\ kgm/s}$	$\mathbf{p_b'} = m_b v_b = ?$

Now, we can start breaking these values into x and y components, and then figure out the totals for both sides.

Before

Since the red ball is going in a straight, horizontal line, we can very easily break it up into x and y components... it's ***all*** x component.

$p_{rx} = 20.52$ kgm/s
$p_{ry} = 0$ kgm/s

Since the blue ball isn't even moving, both of its components are zero.

p_{bx} = 0 kgm/s

p_{by} = 0 kgm/s

We can use these values to figure out the total x component before, and the total y component before the collision...

x total = p_{rx} + p_{bx} = 20.52 kgm/s + 0 kgm/s = 20.52 kgm/s

y total = p_{ry} + p_{by} = 0 kgm/s + 0 kgm/s = 0 kgm/s

These x and y totals will be constant, both before and after the collision...

Before	After
x total = 20.52 kgm/s	x total = 20.52 kgm/s
y total = 0 kgm/s	y total = 0 kgm/s

...they are the same before and after the collision. We can use these totals, along with what we do know about after the collision, to be able to figure out what is missing.

After

We know that after the collision the red ball has 16.2 kgm/s of momentum at an angle of 23° above the horizontal.

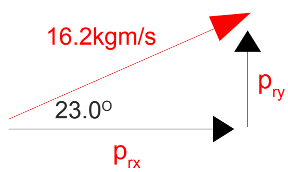

Illustration 4: Red ball after

Break this vector for the red ball into its x and y components by using regular trig.

$$\cos\theta = \frac{adj}{hyp} \qquad \sin\theta = \frac{opp}{hyp}$$

$$\cos 23.0^o = \frac{x}{16.2} \qquad \sin 23.0^o = \frac{y}{16.2}$$

$$p_{rx} = 14.912\, kgm/s \qquad p_{ry} = 6.3298\, kgm/s$$

We know that there is an x total of 20.52, but so far we only know where 14.912 of it can be found. The rest of it must be in the blue ball...

$p_{bx} = p_{x\ total} - p_{rx}$

= 20.52 – 14.912

p_{bx} = 5.608 kgm/s

We can do the same for the y component of the blue ball...

$p_{by} = p_{y\ total} - p_{ry}$

= 0 – 6.3298

p_{by} = -6.3298 kgm/s

The minus sign just means that the y component of the blue ball is pointing down. It makes sense since the two y components **after** must cancel each other to be equal to the y component total from **before** the collision.

We can use the components to figure out the total momentum of the blue ball and the angle it is traveling at...

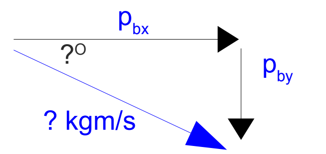

Illustration 5: Blue ball after

$c^2 = a^2 + b^2$
$= 5.608^2 + -6.3298^2$
$c = 8.457$ kgm/s

We want the velocity, not the momentum, of the blue ball...

$p = mv$
$v = p/m = 8.457 / 2.31 = 3.661038961$
$v = 3.66$ m/s

$$\tan\theta = \frac{opp}{adj}$$
$$\tan\theta = \frac{6.3298}{5.608}$$
$$\theta = 48.5^{o}$$

It is important that you make a statement of your FINAL answer at the end of a problem like this, as you've probably been writing numbers all over the place.

The blue ball is traveling at 3.66m/s at an angle of 48.5^{o} below the horizontal.

Example 4: A 1.20 kg red ball moving at 10.0m/s strikes a 2.31 kg blue ball moving at 15.0m/s. If the final velocity of the red ball is 13.5m/s, **determine** the final velocity of the blue ball. Make use of the angles drawn in the following diagram.

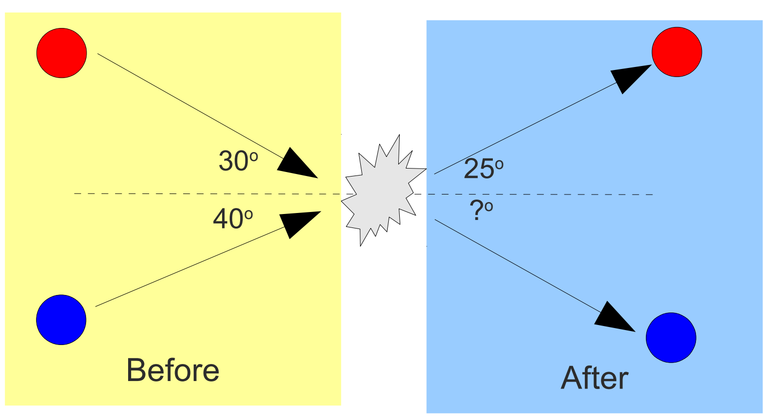

Illustration 6: Sketch of collision for example 4

Since the **total** momentum of the balls before the collision is equal to the **total** momentum of the balls after the collision, we still do this question the same basic way as the last example.

- We will calculate the total momentum of the red and blue balls before the collision by adding their components.
- This resultant is also the resultant of after the collision.
- We will use this, along with the components of the red ball after the collision to figure out the motion of the blue ball.

Before

Do the red ball before the collision…

$p_r = mv = (1.20kg)(10.0m/s)$

$p_r = 12.0$ kgm/s

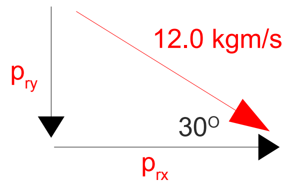

Illustration 7: Red ball before

Figure out the x and y components… you should get x = 10.39230485 kgm/s and y = - 6.00 kgm/s.

Do the blue ball before the collision…

$p_b = mv = (2.31kg)(15.0m/s)$

$p_b = 34.65$ kgm/s

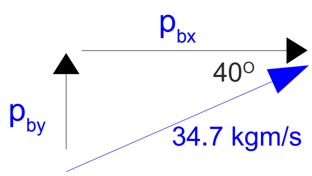

Illustration 8: Blue ball before

Figure out the x and y components… you should get x = 26.54343995 kgm/s and y = 22.27259068 kgm/s.

We will now add the x and y components to get a new resultant.

- The blue ball has a positive y component, but the red ball is negative, so we need to be careful.
- We can just go ahead and add the x components because they point in the same direction.
- After adding the x and y components you should have a new triangle like this.

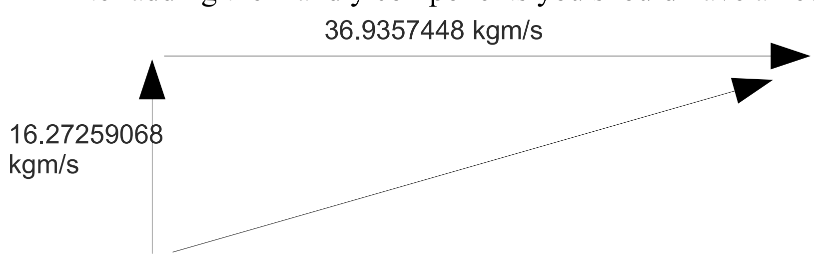

Illustration 9: X and Y totals from before the collision

After

After the collision, all of the momentums have to add up to the same as the triangle drawn above.

- For this question I would suggest you start by calculating the components of the red ball's momentum after the collision.
  - Then you can figure out how much x and y component are "missing."
  - This must come from the blue ball!

The red ball after the collision…

$p_r = mv = (1.20kg)(13.5m/s)$
$p_r = 16.2kgm/s$

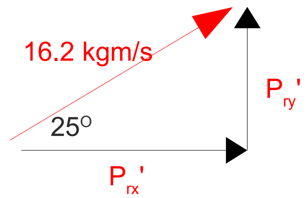

Illustration 10: Red ball after

When you calculate the x and y components you should get $x = 14.68218615kgm/s$ and $y = 6.84641584kgm/s$.

This means that there is a certain amount of x-component unaccounted for…

36.9357448 kgm/s – 14.68218615 kgm/s = 22.25355865kgm/s to the right

We also have some y-component unaccounted for…

- Although we originally assumed the blue ball would be moving down in our diagram, we now know that isn't the case.
  - The red ball had only about 6.84kgm/s pointing up.
  - According to our y-component from before the collision, we need about16.3kgm/s pointing up.
  - So the blue ball must have an upward component of 9.42617484kgm/s upward.

Our blue ball has a diagram after the collision that looks like this…

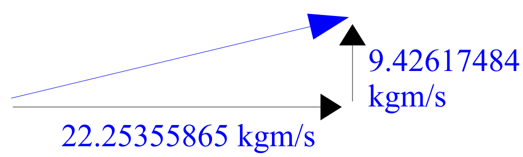

Illustration 11: Blue ball after

- We calculate the hypotenuse as 24.16761562kgm/s.
  - Since the blue ball has a mass of 2.31kg, we can calculate the velocity of the ball as 10.4621 m/s by using p = mv.
  - We use trig to find that the angle is 22.9567° above the horizontal.

Our ***final*** answer is that the blue ball is moving at **10m/s** at an angle of **23°** above the horizontal after the collision.

Homework

p492 #2
p494 #1
p499 #6