

Lesson 13: Voltage in a Uniform Field

Most of the time if we are doing experiments with electric fields, we use parallel plates to ensure that the field is uniform (the same everywhere).

- This carries over to our study of voltage. If we use parallel plates, then we also get a uniform change in potential difference.
 - If a charged particle is released between the plates, it will move on its own from an area of high potential, to an area of low potential.
 - If the charge is moved a distance, d , the change in **electric potential energy** can be found with...

$$W = Fd \Rightarrow \text{which is the same as} \Rightarrow \Delta E_p = Fd$$

Combine that with $V = \frac{\Delta E}{q}$ to get...

$$V = \frac{\Delta E}{q} = \frac{Fd}{q} = \frac{F}{q} d$$

...and we know that $\vec{E} = \frac{F_e}{q}$ so...

$$V = \vec{E} d$$

Although you can certainly use the formula this way, we see it on the data sheet as...

$$|\vec{E}| = \frac{\Delta V}{\Delta d}$$

$|\vec{E}|$ = electric field (N/C)

ΔV = voltage (V)

Δd = distance between locations (m)

This formula is typically applied to situations where parallel plates are being used, since this is the easiest way to have a uniform electric field.

- The direction for the field is still determined by the direction a **positive** test charge would move.
- Also, notice that because of this formula, we can show that the electric field can also be measured in V/m, not just N/C.
 - Try to prove that the units for V/m are equivalent to N/C ... my hint is that you should try to break all the derived units down to base units.

Example 1: We have two parallel plates that are 16.0mm apart.

a) If we want a uniform field of 800 N/C between these plates, **determine** the voltage we must apply to the plates.

$$\begin{aligned}|\vec{E}| &= \frac{\Delta V}{\Delta d} \\ \Delta V &= |\vec{E}| \Delta d \\ \Delta V &= 800(0.0160) \\ \Delta V &= 12.8 \text{ V}\end{aligned}$$

b) **Determine** how much work is done if an alpha particle (*info on your data sheet*) moves from the positive plate to the negative plate.

$$\begin{aligned}V &= \frac{\Delta E}{q} \\ \Delta E &= Vq \\ \Delta E &= 12.8(3.20\text{e-}19) \\ \Delta E &= 4.096\text{e-}18 = 4.10\text{e-}18 \text{ J}\end{aligned}$$

c) **Explain** if this energy is kinetic or potential.

Since the **positive** alpha particle would naturally be repelled by the positive plate and attracted to the negative plate, it will move on its own. This means that **electric potential energy** is turning into **kinetic energy**. The work (ΔE) you just calculated can be thought of as a decrease in **electric potential energy**, or an increase in **kinetic energy**.

All these formulas we've learned in this chapter come together to allow us to do a great variety of questions.

- You might think that the individual formulas are easy, but be careful. There's lots of formulas to keep straight.

Example 2: An alpha particle moves between two parallel plates 2.7e-2 m apart with a potential difference of 130 V between them. **Determine** the force that acts on the particle.

First we need to know how strong the electric field is...

$$|\vec{E}| = \frac{\Delta V}{\Delta d} = \frac{130}{2.7\text{e-}2} = 4814.814815 \text{ N/C}$$

Remember that the unit for electric field can be N/C or V/m.

Then we can figure out the force that is needed...

$$\begin{aligned}\vec{E} &= \frac{F_e}{q} \\ F_e = \vec{E}q &= 4814.814815(3.20\text{e-}19) = 1.540740741\text{e-}15 = 1.5\text{e-}15 \text{ N}\end{aligned}$$

Note that we only round off at the end of the question for sig digs.

Work-Energy Theorem in a Uniform Field

We can do a lot of questions if we keep in mind the work-energy theorem.

- If a charged particle is doing all this moving around, we can use the information we have to figure out how the forces acting on the particle can become **kinetic energy** as it moves.
 - The majority of these questions will involve electrons, so keep in mind that they have a charge of $1.60\text{e-}19\text{ C}$ and a mass of $9.11\text{e-}31\text{ kg}$ (both of which are shown on your data sheet)
 - It is also possible that a question could involve protons, alpha particles, or even some other charged particle that you are given information about.
 - For example, they might tell you that the question deals with a Calcium ion with a mass of $6.65\text{e-}26\text{ kg}$ and a $+2e$ charge ($+3.20\text{e-}19\text{ C}$).

Example 3: An electron is accelerated from rest between two parallel plates, 4.0 cm apart from each other. The voltage between the plates has been set at 20V.

a) **Determine** the velocity of the electron when it reaches the positive plate.

Method 1

Let's go to basics, and just ask ourselves what we can calculate, without worrying about the final answer. We can start with electric field between the plates, since we know the voltage between them and their separation...

$$\vec{E} = \frac{\Delta V}{\Delta d}$$
$$\vec{E} = \frac{20}{0.040}$$
$$\vec{E} = 500\text{ V/m}$$

We are dealing with a uniform electric field, so the force will be constant anywhere in the field. We should probably calculate that force...

$$\vec{E} = \frac{F_e}{q}$$
$$F_e = \vec{E} q$$
$$F_e = 500(1.60\text{e-}19)$$
$$F_e = 8\text{e-}17\text{ N}$$

Once we have the force (and we'll treat it as net force since we don't have any other forces to keep track of), we can go back to Physics 20!

$$a = \frac{F_{NET}}{m}$$
$$a = \frac{8\text{e-}17}{9.11\text{e-}31}$$
$$a = 8.7815587\text{e}13\text{ m/s}^2$$

Finally, we calculate the final velocity of the electron as it reaches the positive plate...

$$v_f^2 = v_i^2 + 2ad$$

$$v_f^2 = 0^2 + 2(8.7815587 \times 10^{13})(0.040)$$

$$v_f = 2.65052 \times 10^6 = 2.7 \times 10^6 \text{ m/s}$$

Notice that this is about 1% the speed of light. If we were in a university level course, we would probably have to redo our calculations, taking into account Einstein's Special Theory of Relativity. At these sorts of speeds, lots of weird stuff starts to happen. You don't need to worry about it in this course.

Method 2

We know that the electron is going to be picking up a lot of kinetic energy as it is attracted to the positive plate and repulsed by the negative plate. We can calculate this **change in energy** (the **work** done on the electron) this way...

$$V = \frac{\Delta E}{q}$$

$$\Delta E = Vq$$

$$\Delta E = 20(1.60 \times 10^{-19})$$

$$\Delta E = 3.2 \times 10^{-18} \text{ J}$$

Since the electron started from rest, any energy it gained must be in the form of kinetic energy. We can figure out the velocity of the electron this way...

$$E_k = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$v = \sqrt{\frac{2(3.2 \times 10^{-18})}{9.11 \times 10^{-31}}}$$

$$v = 2.7 \times 10^6 \text{ m/s}$$

b) You have calculated the maximum velocity for the situation where the electron has accelerated all the way from the negative to the positive plate, going through the entire potential difference. **Determine** the electron's velocity when it has only moved 1.0 cm away from the negative plate.

Method 1

If we did method 1 for the first question, we can reuse a bunch of our numbers. The electric field, electrostatic force, and acceleration of the electron are all the same! The only difference is that the electron only moves 1.0 cm through the field, instead of all 4.0 cm.

$$v_f^2 = v_i^2 + 2ad$$

$$v_f^2 = 0^2 + 2(8.78 \times 10^{13})(0.010)$$

$$v_f = 1.325142 \times 10^6 = 1.3 \times 10^6 \text{ m/s}$$

Method 2

Start with the electric field we calculated originally, 500 V/m. This really mean that for every metre the charge moves through the field it would go through 500 V. Our electron is only going through a fraction of that...

$$\begin{aligned}|\vec{E}| &= \frac{\Delta V}{\Delta d} \\ \Delta V &= |\vec{E}| \Delta d \\ \Delta V &= 500(0.010) \\ \Delta V &= 5.0\text{V}\end{aligned}$$

Knowing the voltage (potential difference) the electron has passed through and its charge, we can figure out the change in energy...

$$\begin{aligned}V &= \frac{\Delta E}{q} \\ \Delta E &= Vq \\ \Delta E &= 5(1.60\text{e-}19) \\ \Delta E &= 8\text{e-}19\text{ J}\end{aligned}$$

This change in energy is electric potential lost and ***kinetic energy gained!*** (Work-energy theorem)

$$\begin{aligned}E_k &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2E_k}{m}} \\ v &= \sqrt{\frac{2(8\text{e-}19)}{9.11\text{e-}31}} \\ v &= 1.32526\text{e}6 = 1.3\text{e}6\text{ m/s}\end{aligned}$$

Method 3

We know that the electric potential (voltage) is spread out evenly over the whole space between the plates. So, we could set up a ratio based on the fact that the electric field is also a constant all the way in between the plates.

$$\begin{aligned}|\vec{E}| &= \frac{\Delta V}{\Delta d} \\ \frac{\Delta V_1}{\Delta d_1} &= \frac{\Delta V_2}{\Delta d_2} \\ \frac{20}{0.040} &= \frac{\Delta V_2}{0.010} \\ \Delta V_2 &= 5\text{ V}\end{aligned}$$

Now that we have the voltage, we can do the rest of the question the same as method 2.

Example 4: An electron is moving to the right at $1.4\text{e}6$ m/s. It enters an area perpendicular to an electric field that points to the left. The potential difference between the plates is 5.0 V. **Determine** the velocity of the electron as it exits the electric field.

Super important! Since the electric field points to the **left**, and that is based on what it would do to push around a **positive** test charge, the **electron** will be pushed to the **right**! This means that the electron will accelerate to a faster speed.

The electron already had kinetic energy before it entered the field, since it was already moving...

$$E_k = \frac{1}{2}mv^2$$

$$E_k = 0.5(9.11\text{e-}31)(1.4\text{e}6)^2$$

$$E_k = 8.9278\text{e-}19\text{ J}$$

The electric field will add energy to the electron (according to Work-Energy Theorem)...

$$V = \frac{\Delta E}{q}$$

$$\Delta E = qV$$

$$\Delta E = 1.60\text{e-}19(5.0)$$

$$\Delta E = 8\text{e-}19\text{ J}$$

So, we can figure out how much kinetic energy the electron has as it comes out the other side...

$$\Delta E = E_k' - E_k$$

$$E_k' = \Delta E + E_k$$

$$E_k' = 8\text{e-}19 + 8.89278\text{e-}19$$

$$E_k' = 1.69278\text{e-}18\text{ J}$$

Finally, we can use this final kinetic energy to determine the final velocity of the electron...

$$E_k' = \frac{1}{2}mv'^2$$

$$v' = \sqrt{\frac{2E_k'}{m}}$$

$$v' = \sqrt{\frac{2(1.69278\text{e-}18)}{9.11\text{e-}31}}$$

$$v' = 1927773 = 1.9\text{e}6\text{ m/s}$$

Homework

p568 #1, 2
 p571 #1
 p573 #2
 p574 Example 11.12