Lesson 13g: Resistors in Series and Parallel Circuits

Any path along which electrons can flow is a circuit.

- For a continuous flow of electrons, there must be a complete circuit with no gaps. A
- A gap is usually a switch that can be ***closed*** (on) to allow electron flow or ***open*** (off) to stop electron flow.
- If I wanted to draw a **schematic diagram** (aka **circuit diagram**) of a resistor powered by a battery that can be turned off and on, it might look like Illustration 1.
  - Note that the placement of the switch (SW1) near the positive terminal (what we would consider the end of the electron flow path) does not matter. A switch anywhere in the circuit will have the same effect of stopping current flow.
  - Switches are shown in the open (off) position so they are easier to spot in the diagram. In a closed (on) position, the current would be able to flow through the whole circuit.

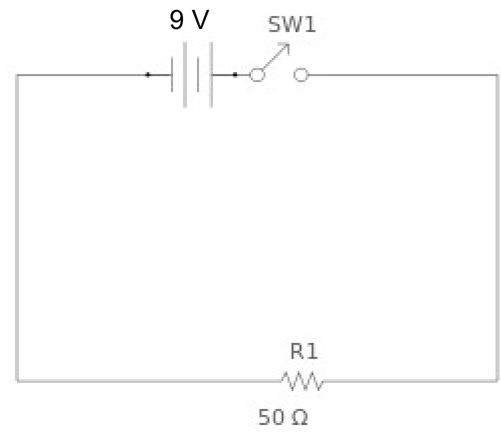


Illustration 1: A simple schematic diagram of a battery connected to a switch and resistor.

Most circuits used in modern devices will have several components, hooked up in a variety of combinations. All of these circuits can be broken into two main categories, **series** and **parallel**.

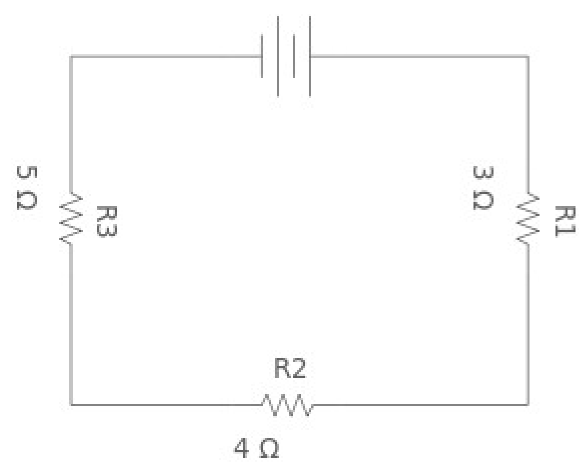
- In all circuits, going through a battery increases voltage, while going across a resistor decreases voltage (a voltage "drop").
- As you go around any single pathway the total voltage drops across all resistors must equal the voltage from the battery.

Series Circuits

In a **series circuit** there is only a single pathway for electron flow between the terminals of the battery. If you can trace only one pathway with your finger, it's a **series circuit**.

When the switch is closed, a current exists almost immediately in all three resistors.

- The current does not "pile up" at any of the resistors, nor does it go at different rates in different parts of the circuit.
  - If you are in line to get into a movie, it doesn't matter where you are standing; the whole line moves at the same speed. Everyone is affected by trying to get by the ticket window.



- For this reason we can say that **the current is constant everywhere in a series circuit**.

A break anywhere in the path results in an open circuit, and the flow of electrons stops.

- Burning out one resistor or simply opening a switch could cause such a break.

The current has to flow through all these resistors, so overall we increase the resistance of a circuit when we place resistors in series.

- As mentioned earlier the total drops in voltage must equal the voltage from the battery along a single path (exactly what a series circuit is). Since V = IR we can say...

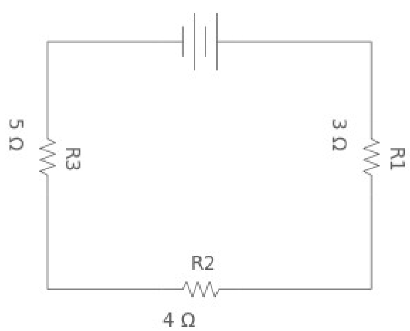
$$V_T = V_1 + V_2 + V_3$$
$$IR_T = IR_1 + IR_2 + IR_3$$
$$R_T = R_1 + R_2 + R_3$$

- The "T" refers to the total, or overall equivalent, voltages and resistances.
- We can safely cancel "I" since the current is the same everywhere in a series circuit.
- This means that if we take all the resistances and add them up, we get a single equivalent resistance.
    - If we do this we can use V =IR since we will now have only one voltage and one resistor.

To see how all this (and more) can work, let's actually start figuring our numbers for the circuit shown earlier.

- We will do this while keeping track of our numbers on a "VIR" chart, as shown.
- You'll also see that we will be using Ohm's Law (V = IR) over and over again!

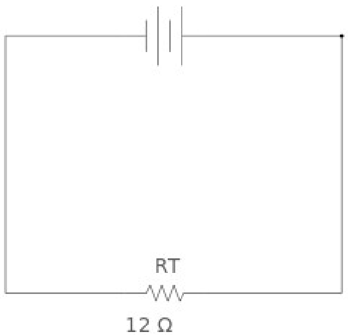
Example 1: **Determine** the voltage drops and current flowing through each part of the circuit shown here. The battery is 9 volts. Do not worry too much about sig digs for this example.



	V	I	R
R_1			**3**
R_2			**4**
R_3			**5**
R_T	**9**		

The VIR chart just quickly summarizes everything we know up to this point, the overall voltage from the battery and the three resistors resistances.
The first thing we should do is "scrunch" the circuit to find the equivalent resistance of the three resistors. We do this by simply adding the three resistances and redrawing the diagram.

$$R_T = R_1 + R_2 + R_3$$
$$R_T = 3 + 4 + 5$$
$$R_T = 12\,\Omega$$



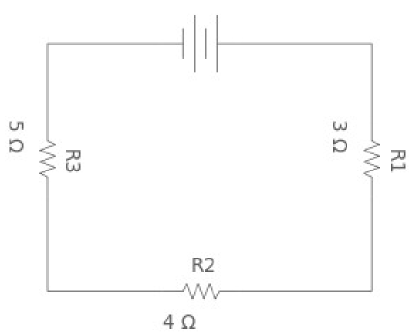
	V	I	R
R_1			3
R_2			4
R_3			5
R_T	9		**12**

Notice in the last column we have two of the three numbers. We can use V = IR to calculate the current, which is the overall current flowing in my scrunched circuit diagram.

$$I = \frac{V}{R_T}$$
$$I = \frac{9}{12}$$
$$I = 0.75\ A$$

	V	I	R
R_1			3
R_2			4
R_3			5
R_T	9	**0.75**	12

Now the great news is that even if we "unscrunch" the diagram back to what it was originally, the current we just calculated will still be the same everywhere. This is because current is constant everywhere in a series circuit.



	V	I	R
R_1		**0.75**	3
R_2		**0.75**	4
R_3		**0.75**	5
R_T	9	0.75	12

Since we know the current flowing through each individual resistor, we can calculate the voltage drop on each one.

$$V_1 = I R_1 \qquad V_2 = I R_1 \qquad V_3 = I R_1$$
$$V_1 = 0.75(3) \qquad V_2 = 0.75(4) \qquad V_3 = 0.75(5)$$
$$V_1 = 2.25\ V \qquad V_2 = 3\ V \qquad V_3 = 3.75\ V$$

Fill in the last three columns now.

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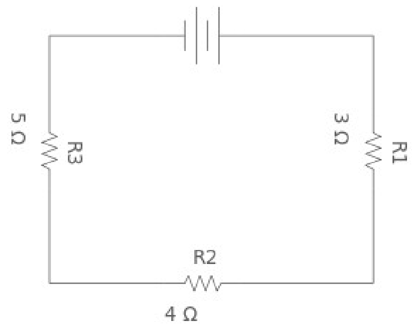


	V	I	R
R_1	**2.25**	0.75	3
R_2	**3**	0.75	4
R_3	**3.75**	0.75	5
R_T	9	0.75	12

Notice, as a double check, that if you add the voltage drops across the three resistors, you will get the same number (9V) as the voltage from the battery.
Using the values on the chart, you could now say what the current and voltage drops are at any point along the circuit.

Parallel Circuits

In a parallel circuit there are several pathways for electron flow between the terminals of the battery. If you can trace more than one pathway with your finger, it's a parallel circuit.
When the switch is closed, a current exists almost immediately in all three resistors.

- When the current gets to a junction (a point where the wires split), the current can go in more than one direction.
    - This means that the current keeps splitting up.
    - Current will be greatest near the battery on the main branch (highlighted in the picture in red), since it hasn't split up.
    - On each of the branches with resistors the current will be smaller.
    - If we take all the currents on all the branches and add them, we get the current on the main branch.

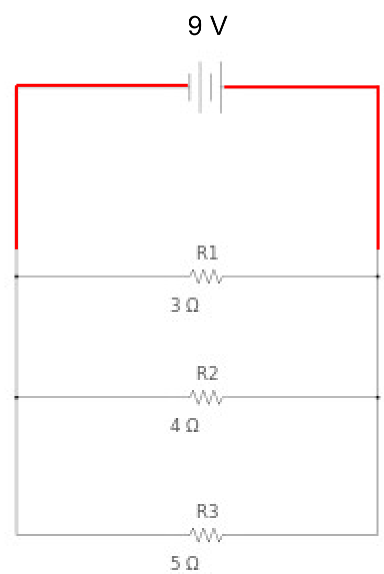


Illustration 3: A parallel circuit with three resistors.

- Each resistor has its own direct connection to both terminals of the battery.
    - For this reason **each resistor has the same full voltage from the battery in a parallel circuit**.

As long as it is not on the main branch, a break anywhere in the circuit does not affect the other resistors, since they have their own paths to the battery.

- Putting a switch on the main branch would affect all the resistors.

Because charge is conserved, the current flowing into a junction must equal the current flowing out.

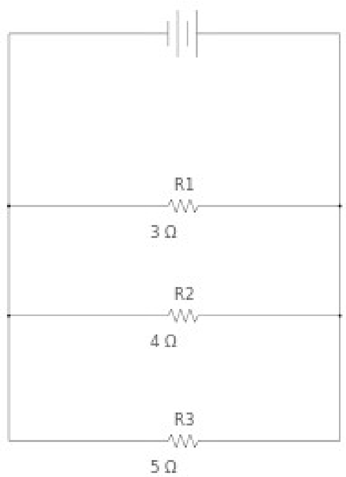
- Kind of like the way we played around with formulas for series circuits above, we can say that the current on the main branch is I_T and the current on each branch with the resistors are I_1 , I_2 , and I_3 .
- The voltage is constant everywhere in the circuit.

$$I_T = I_1 + I_2 + I_3$$
$$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

You can NOT just say "Oh, now I'll take the inverse of this formula and just get $R_T = R_1 + R_2 + R_3$... they are ***NOT*** mathematically equivalent.

- Also notice that this means that adding more resistors in parallel actually ***decrease*** the overall resistance!

Example 2: **Determine** the voltage drops and current flowing through each part of the circuit shown here. The battery is 9 volts. Do not worry too much about sig digs for this example.



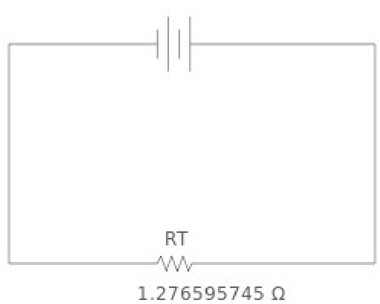
	V	I	R
R_1			**3**
R_2			**4**
R_3			**5**
R_T	**9**		

The VIR chart looks like it did before. The difference will be when we scrunch the parallel resistors down to one equivalent resistor.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{R_T} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$
$$R_T = 1.276595745\,\Omega$$

Warning!
Remember to take the inverse at the very end of this calculation in order to get the equivalent resistance, instead of its inverse!

We really need to watch out for rounding off numbers here. Keep the originals written down and use them in all your calculations.



	V	I	R
R_1			3
R_2			4
R_3			5
R_T	9		**1.276595745**

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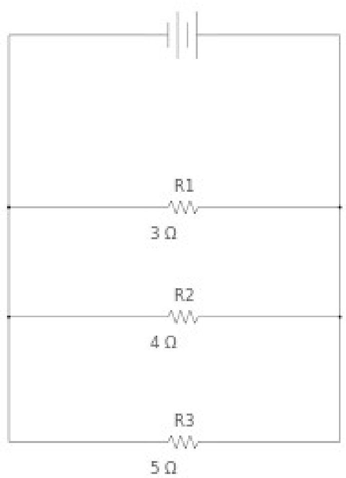
Just like before, let's calculate the current in this equivalent circuit.

$$I_T = \frac{V_T}{R_T}$$
$$I_T = \frac{9}{1.276595745}$$
$$I_T = 7.05\,A$$

	V	I	R
R_1			3
R_2			4
R_3			5
R_T	9	**7.05**	1.276595745

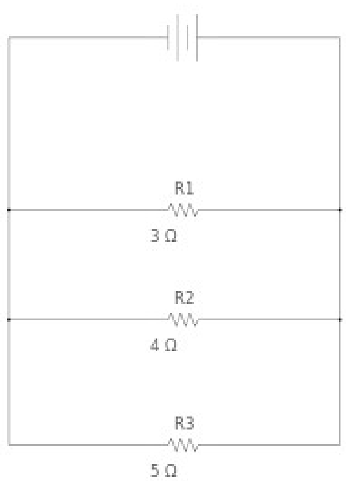


In a parallel circuit, the voltage is the same everywhere, so when we unscrunch the circuit we can say the current on each resistor is the same.

	V	I	R
R_1	**9**		3
R_2	**9**		4
R_3	**9**		5
R_T	9	7.05	1.276595745

So now we can solve for the missing currents in each of the individual branches with the individual resistors,

$$I_1 = \frac{V}{R_1} \qquad I_2 = \frac{V}{R_2} \qquad I_3 = \frac{V}{R_3}$$
$$I_1 = \frac{9}{3} \qquad I_2 = \frac{9}{4} \qquad I_3 = \frac{9}{5}$$
$$I_1 = 3\,A \qquad I_2 = 2.25\,A \qquad I_3 = 1.8\,A$$



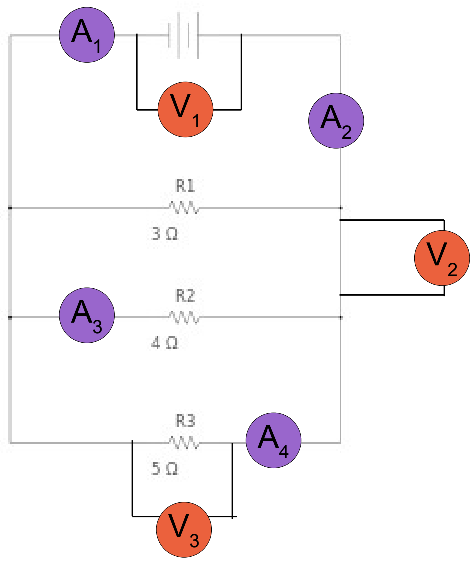
	V	I	R
R_1	9	**3**	3
R_2	9	**2.25**	4
R_3	9	**1.8**	5
R_T	9	7.05	1.276595745

As a double check, we notice that all the currents in the separate branches add up to the current on the main branch.

On most diagrams you will find that they ask for specific **currents** and **voltage drops** by drawing numbered **ammeters** and **voltmeters** on the diagram.

- **Ammeters** measure **current** in **amps,** and must be placed in the circuit so the current can go through them.
- **Voltmeters** measure **potential difference** in **volts**, and must be placed parallel to whatever they measure (since we want to measure the potential ***difference*** across the device).

Example 3: The circuit you just analyzed has had some ammeters and voltmeters placed on it. **Identify** the reading each would give.



Reading from our chart...

A_1 = 7.05 A (it's on the main branch)
A_2 = 7.05 A (also on the main branch)
A_3 = 2.25 A (on the R_2 branch)
A_4 = 1.8 A (on the R_3 branch)

V_1 = 9 V (voltage increase across battery)
V_2 = zero (no voltage increase or drop!)
V_3 = 9 V (voltage across R_3)

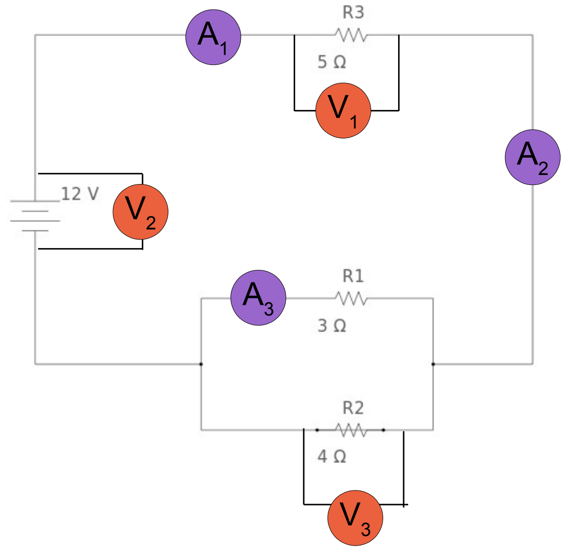
Combination Circuits

Combination circuits are more realistic, since they are circuits with series and parallel parts together.

- You'll spot these by noticing that at least a couple resistors are in parallel, but that if you scrunch them, you'll end up with a series circuit for the other resistors.
- As a general rule, you should try to scrunch the parallel parts first, then the series.
  - Do NOT try to skip steps... it will lead to errors.
  - You might have to scrunch resistors in series ***within*** a parallel circuit first!

Example 4: **Determine** the ammeter and voltmeter readings for the following schematic.

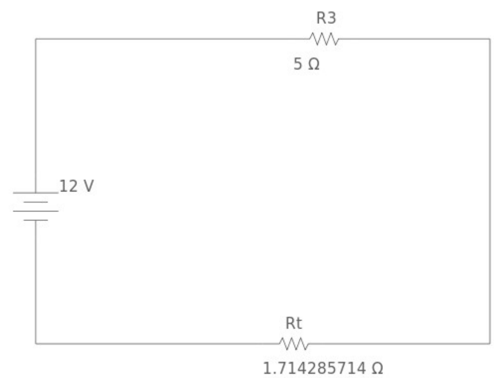
Make a VIR chart, fill in everything you can, and then start to scrunch.



	V	I	R
R_1			3
R_2			4
R_3			5
R_T	12		

We figure out the equivalent resistance of R_1 and R_2 first, since they are in parallel to each other. We'll call it R_t.

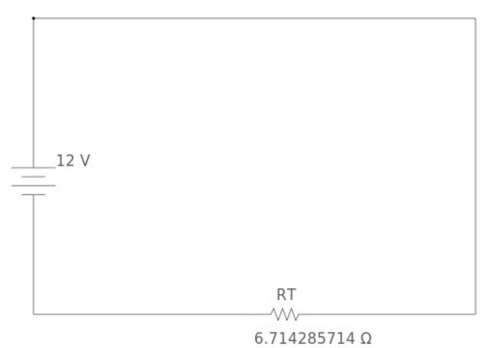
$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$\frac{1}{R_t} = \frac{1}{3} + \frac{1}{4}$$
$$R_t = 1.714285714\,\Omega$$



That's really just sort of an in between equivalent resistance, since we now have to scrunch once more to find the equivalent resistance of these two in parallel.

$$R_T = R_t + R_3$$
$$R_T = 1.714285714 + 5$$
$$R_T = 6.714285714\,\Omega$$

	V	I	R
R_1			3
R_2			4
R_3			5
R_T	12		6.714285714



Ok, you know what's coming. Start using Ohm's Law to calculate anything you can, in this case, the overall current.

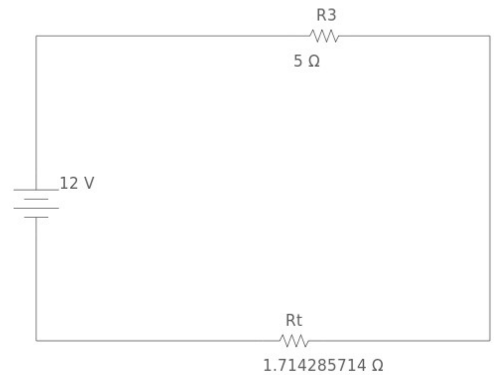
	V	I	R
R_1			3
R_2			4
R_3			5
R_T	12	1.787234043	6.714285714

$$I=\frac{V}{R}$$
$$I=\frac{12}{6.714285714}$$
$$I=1.787234043\,A$$

Now be careful! Only unscrunch one step back, so it is just a boring series circuit with two resistors. That way you can still say “current same everywhere” and calculate the voltage drop on the two resistors.

$$V_3=I\,R_3$$
$$V_1=1.787234043(5)$$
$$V_1=8.936170213\,V$$

$$V_t=I\,R_t$$
$$V_t=1.787234043(1.714285714)$$
$$V_t=3.063829787\,V$$

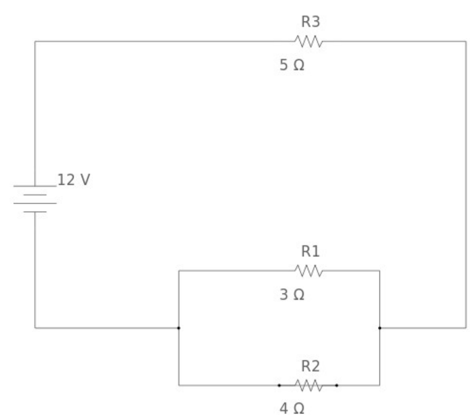


	V	I	R
R_1	3.063829787		3
R_2	3.063829787		4
R_3	8.936170213		5
R_T	12	1.787234043	6.714285714

You might be saying “Yikes! How come you put the voltage drop from our 'in-between' equivalent resistor R_t for both R_1 and R_2?” Thats because when it gets unscrunched the two resistors are in parallel, and voltage is constant in parallel!

Anyway, we now have enough info to calculate the other currents using Ohm's Law... I'll leave it up to you to double check my math here.

	V	I	R
R_1	3.063829787	1.021276596	3
R_2	3.063829787	0.765957447	4
R_3	8.936170213	1.787234043	5
R_T	12	1.787234043	6.714285714



Notice a couple things?

1. The current in R_3 is the same as the overall current. This is because, technically, its on the main branch of the circuit. The current splits to go through R_1 and R_2; those two currents add up to the overall current.
2. The three voltage drops do not add up to the overall voltage from the battery. That's because you can not go through R_1 and R_2 at the same time... you have to pick a path through one or the other. When you do, you'll see the path through one of them, along with R_3, is equal to 12 V.

The (slightly rounded off) readings on the ammeters and voltmeters are...

A1 = 1.79 A	V1 = 8.94 V
A2 = 1.79 A	V2 = 12 V
A3 = 1.02 A	V3 = 3.06 V