

Thermodynamics Lesson 1: Kinetic Theory of Ideal Gases

The Theory

The kinetic theory of ideal gases is one of those rare things in physics... it makes sense!

- Basically, from the time you started to learn about gases in elementary school, you were learning about the basic postulates (ideas) of the kinetic theory of ideal gases.
- To remember the postulates given here, try to think of a sealed container holding a gas. Pretty much everything that is said here is the sort of stuff that should pop into your head.
 1. The container holds a *huge* number of molecules of the gas.
 2. The space between the molecules is *huge* in comparison to the size of the molecules themselves.
 3. The individual molecules are in constant, random motion.
 4. The molecules can not affect each other at a distance (no gravitational or electrostatic forces on each other).
 5. All collisions (molecule-on-molecule or molecule-on-container wall) are **elastic** (*no loss of kinetic energy*).*

* More on this in Physics 30

Root-Mean-Square Speed

Since a container of gas has so many molecules bouncing around all the time (at pretty high velocities!), we can understand why there is pressure against the sides of the container.

- As the gas molecules hit the sides of the container, they will bounce off without losing any of their kinetic energy.
- Because of Newton's 3rd Law, as much as the side of the container exerts a force on the molecule to reflect it inwards, the molecule exerts an equal but opposite force pushing the side outwards.
- The molecules will all be moving at different speeds, so if we talk about a large group of molecules we should talk about the *average* speed.
 - We need to take the average of the speed of the molecules, but the formula for kinetic energy has velocity squared.
 - For this reason we use a special value for the value of the “speed squared” called the **root-mean-square (rms) speed**.
 - The velocity of the gas molecules is found using...

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_b T}{\mu}}$$

v_{rms} = root-mean-square speed

R = universal gas constant (8.31 J/(mol•K))

T = Temperature (K)

M = molar mass (**kg**/mol)

k_b = Boltzmann's Constant (1.38e-23 J/K)

μ = mass of a molecule (kg)

Warning!

Temperatures must always be measured in Kelvin, where...

$$^{\circ}\text{C} + 273 = \text{K}$$

...and molar mass must be measured in **kg/mol**, not g/mol.

- Notice how the speed is directly related to the square root of the temperature.
 - This makes sense since we assume higher temperature gases have faster moving molecules.
- We also see that the speed is inversely related to the square root of the molar mass of a molecule.
 - Again logical, since a heavier molecule will move more slowly.

Example 1: Determine the rms speed of oxygen molecules (O_2) in a room at $20^\circ C$.

No matter what version of the formula we choose to use, we must first convert the temperature of the air from degrees Celsius to Kelvin...

$$20^\circ C + 273 = 293 \text{ K}$$

Method 1

If we are going to use the first version of the formula, we need the molar mass (M) of oxygen.

- On the periodic table it shows that oxygen is 16.00 g/mol.
- Oxygen is **diatomic**, so on its own it is always O_2 and has a mass for each molecule of 32.00 g/mol.
- We must change this to kilograms, so divide by 1000 to get $M = 0.03200 \text{ kg/mol}$.

Now use the formula...

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} = \sqrt{\frac{3(8.31)(293)}{0.03200}}$$

$$v_{rms} = 478 \text{ m/s}$$

Method 2

For this method we need to know the mass of a **single molecule** of oxygen in atomic mass units (u).

- Just like in Method 1, we figure out that oxygen molecules (O_2) have a molar mass of 32.00 g/mol.
- That means that the mass of O_2 is 32.00 u (it's just the same number!).
- The AP data sheet gives the conversion factor to find kilograms, so we'll do that next...

$$32.00 \text{ u} (1.66\text{e-}27 \text{ kg/u}) = 5.312\text{e-}26 \text{ kg}$$

Warning!

Atomic mass units (u) is a way to measure the mass of very small things. You'll learn more about it in Physics 30. For now, just remember that the molar mass in g/mol is equal to the atomic mass units.

And now we can use the second version of the formula for rms speed...

$$v_{rms} = \sqrt{\frac{3k_b T}{\mu}}$$

$$v_{rms} = \sqrt{\frac{3(1.38\text{e-}23)(293)}{5.312\text{e-}26}}$$

$$v_{rms} = 478 \text{ m/s}$$

Average Translational Kinetic Energy

Since all these molecules of gas are moving around so much, it's reasonable to suggest that they have kinetic energy.

- In statistics it is hard to say anything about individuals, but easy to make predictions about large groups.
- That's why we must talk about the average kinetic energy of these molecules, usually referred to as the **average translational kinetic energy**.
 - The “translational” part refers to how the molecule shifts as it moves from place to place, but does not take into account other motion like the molecule spinning on its own axis.
 - Although this happens in three dimensions, we only focus on the component that directly hits the wall of the container.
 - The formula is based on some really fun relationships between kinetic energy and pressure, and comes out as...

$$K_{avg} = \frac{3}{2} k_b T$$

K_{avg} = average translational kinetic energy (J)

k_b = Boltzmann's Constant (1.38×10^{-23} J/K)

T = Temperature (K)

If you are interested in seeing where this formula comes from
(although it is **NOT** required) you can check out this video:
<https://www.youtube.com/watch?v=UMXSNjjUVt4>

Example 2: Determine the average kinetic energy of oxygen molecules (O_2) in a room at 20°C .

Method 1

If you had just figured out the rms speed of the oxygen molecules like we did in *Example 1*, you could just use the regular kinetic energy formula. You will need the mass of the oxygen molecule in kilograms that we figured out in *Example 1 Method 2*.

$$E_k = \frac{1}{2} mv^2$$

$$E_k = \frac{1}{2} (5.312 \times 10^{-26}) (478^2)$$

$$E_k = 6.07 \times 10^{-21} \text{ J}$$

Method 2

You could also use the formula we just learned...

$$K_{avg} = \frac{3}{2} k_b T$$

$$K_{avg} = \frac{3}{2} (1.38 \times 10^{-23}) (293)$$

$$K_{avg} = 6.07 \times 10^{-21} \text{ J}$$

This method reveals that the average kinetic energy of the molecules of a gas does **not** depend on the gas in question, only on the temperature.

This means that the temperature of a gas is a direct representation of the average energy of the molecules of that gas.

For example, in a room filled with regular air...
the heavier gas molecules (like CO₂) will be moving slower,
and lighter gas molecules (like O₂) will move faster,
and yet have the same average kinetic energy if they share the same temperature.

This also means that the pressure in a container is related to its temperature because of the energy of the individual molecules in the container hitting its sides.