# Coulomb's Law

## Charles Augustin de Coulomb


Figure 1: Coulomb

Before getting into all the hardcore physics that surrounds him, it's a good idea to understand a little about Coulomb.

- He was born in 1736 in Angoulême, France.
- He received the majority of his higher education at the Ecole du Genie at Mezieres (sort of the French equivalent of universities like Oxford, Harvard, etc.) from which he graduated in 1761.
- He then spent some time serving as a military engineer in the West Indies and other French outposts, until 1781 when he was permanently stationed in Paris and was able to devote more time to scientific research.

Between 1785-91 he published seven memoirs (papers) on physics.

- One of them, published in 1785, discussed the inverse square law of forces between two charged particles. This just means that as you move charges apart, the force between them starts to decrease faster and faster (exponentially).
- In a later memoir he showed that the force is also proportional to the product of the charges, a relationship now called "**Coulomb's Law**".
- For his work, the unit of electrical charge is named after him. This is interesting in that Coulomb was one of the first people to start creating the metric system.
- He died in 1806.

## The Torsion Balance

When Coulomb was doing his original experiments he decided to use a torsion balance to measure the forces between charges.

- You already learned about a torsion balance in Physics 20 when you discussed Henry Cavendish's experiment to measure the value of "G" , the universal gravitational constant. Review Cavendish's work in the Lesson 35: Universal Gravitation notes from Physics 20 if you need to.
- Coulomb was actually doing his experiments about 10 years before Cavendish.
- He set up his apparatus as shown below with all spheres charged the same way.
    - He charged one of the free moving spheres.
    - He then touched it to the other free moving sphere (charging it by conduction).
    - Each of the free moving spheres was then touched to one of the spheres on the rod.
    - Although he didn't know the actual charge on any particular sphere, Coulomb did know that each was equal.
    - Coulomb also altered the experiment by using spheres of different sizes so he could get the amounts of charge in different ratios.

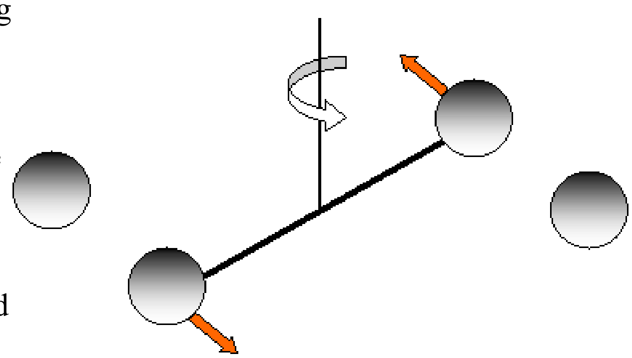


Figure 2: The Torsion Balance

Because like charges repel, the spheres on the torsion balance twist away from the other balls.

- By knowing the distance between the balls, the force needed to twist them (the torque from which the torsion balance gets it name), and the charges on the balls, he could figure out a formula.

After all this work Coulomb finally came up with a formula that could be used to calculate the force between any two charges separated by a distance…

$$F_e = \frac{k q_1 q_2}{r^2}$$

$F_e$ = Force (N)
q = Charge (C)
r = distance between the charges (m)
k = 8.99e9 $Nm^2/C^2$

Notice that this formula looks almost identical to the formula for Universal Gravitation…

$$F_g = \frac{G m_1 m_2}{r^2}$$

**Example 1**: One charge of 2.0 C is 1.5m away from a –3.0 C charge. **Determine** the force they exert on each other.

$$F_e = \frac{k q_1 q_2}{r^2}$$

$$F_e = \frac{8.99\text{e}9 \, x \, 2.0 \, x - 3.0}{1.5^2}$$

$$F_e = -2.4\text{e}10 \, N$$

The **negative** sign just means that one charge is positive, the other is negative, so there is an **attractive** force between them.

Wow, that's a lot of force!

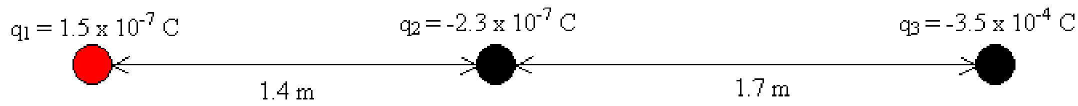
- Just so you know, it is ***VERY*** rare to find charges as big as this.
- In the lab or in everyday life charges are usually in the range of about $10^{-6}$C (1 mC).

## Multiple Charges in One Dimension (Linear)

Things get a bit more interesting when you start to consider questions that have more than two charges.

- You will almost always deal with three charges in these linear problems.
- In the following example you have three charges lined up and are asked to calculate the net force acting on one of them.
- Do one step at a time, and then combine the answers at the end.

**Example 2**: The following three charges are arranged as shown. **Determine** the net force acting on the charge on the far right ($q_3$ = charge 3).




Calculate the force between one pair of charges, then the next pair of charges, and so on until you have calculated all the possible combinations for that particular question. Remember, if you've calculated the force of $q_1$ on $q_2$, then you also know the force of $q_2$ on $q_1$ ... they're the same!

*Step 1: Calculate the force that charge 1 exerts on charge 3...*
It does **NOT** matter that there is another charge in between these two… ignore it! It will not effect the calculations that we are doing for these two. Notice that the total distance between charge 1 and 3 is 3.1 m , since we need to add 1.4 m and 1.7 m .

$$F_e = \frac{k q_1 q_3}{r^2} = \frac{8.99\text{e}9 \; x \; 1.5\text{e-}7 \; x - 3.5\text{e-}4}{3.1^2} = -4.9\text{e-}2 \; N$$

The **negative** sign just tells us the charges are **opposite**, so the force is **attractive**. Charge 1 is pulling charge 3 to the left, and vice versa. ***Do not automatically treat a negative answer as meaning "to the left" in this formula!!!*** Since all I care about is what is happening to charge 3, all I really need to know from this is that charge 3 feels a pull towards the left of 4.9e-2 N.

*Step 2: Calculate the force that charge 2 exerts on charge 3...*
Same thing as above, only now we are dealing with two negative charges, so the force will be repulsive.

$$F_e = \frac{k q_2 q_3}{r^2} = \frac{8.99\text{e}9 \; x - 2.3\text{e-}7 \; x - 3.5\text{e-}4}{1.7^2} = +2.5\text{e-}1 \; N$$

The **positive** sign tells you that the charges are either **both negative or both positive**, so the force is **repulsive**. I know that charge 2 is pushing charge 3 to the right with a force of 2.5e-1 N.

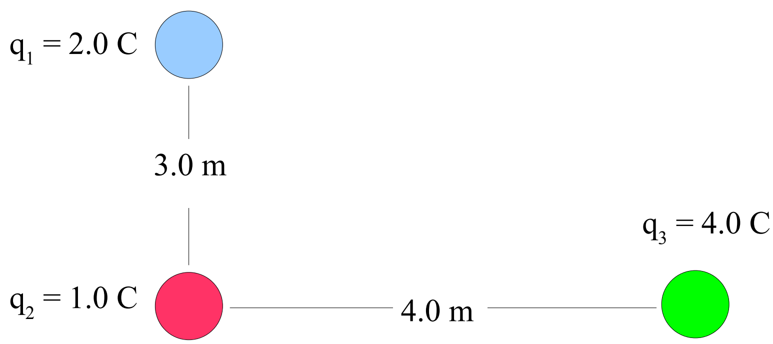
*Step 3: Add you values to find the net force.*
We now need to add the two values from above, being careful about directions. We have a 4.9e-2 N force pulling charge 3 to the left, which just happens to be the direction we usually call negative, so we'll put the negative sign on it. We also have a 2.5e-1 N force pushing to the right. Again, we just happen to be lucky that the sign on the force (positive) agrees with the fact that we usually say that to the right is positive.

**$F_{NET}$ = -4.9e-2 N + 2.5e-1 N = 2.0e-1 N**

## Multiple Charges in 2 Dimensions

Doing questions with charges in multiple dimensions are the same as the question you did above. You just need to be careful about directions and use vectors to figure out the problem.

**Example 3**: Three charges are arranged in a right angle triangle as the following diagram shows. **Determine** the force on $q_2$ .

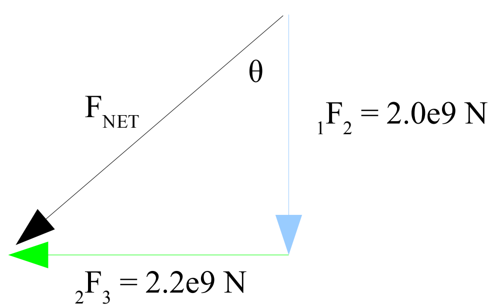



We need to start by calculating the individual forces on $q_2$ by each of the other charges. These must be calculated individually.

$$_1F_2 = \frac{k q_1 q_2}{r^2} = \frac{8.99e9 \, x \, 2.0 \, x \, 1.0}{3.0^2} = 1997777777 = 2.0e9 \, N$$

$$_2F_3 = \frac{k q_2 q_3}{r^2} = \frac{8.99e9 \, x \, 1.0 \, x \, 4.0}{4.0^2} = 2247500000 \, N = 2.2e9 \, N$$

All of the charges are positive, so all of the forces are repulsive. That means that $_1F_2$ is a force that is pushing $q_2$ down, and $_2F_3$ is a force pushing $q_2$ to the left.




It's easy enough to calculate $F_{NET}$ using Pythagoras, and figure out the angle using trig.

$c^2 = a^2 + b^2$
$= (2.0e9)^2 + (2.2e9)^2$
$c = 3.0e9$ N

$\tan\theta = opp / adj$
$\tan\theta = 2.2e9 \, N / 2.0e9 \, N$
$\theta = 48^O$