Electric Fields & Electric Field Lines

Electric Fields

Just like the force due to gravity, the force due to electric charges can act over great distances.

- Keep in mind that most forces we deal with in everyday life are “contact forces”… objects touch each other directly in order to exert a force on each other.
  - e.g. tennis racket hits a tennis ball
- The idea of even considering forces that could happen without anything touching (forces at a distance) was very difficult for early scientists to accept, from Aristotle to Newton.

Then the British scientist Michael Faraday came up with the idea of a “field”.

- A field is sometimes defined as a sphere of influence. An object within the field will be affected by it.
  - Think of how you talk about countries in social studies... large, powerful nations can have an influence on nearby countries. Usually as you get further away from the powerful nation, the influence they have on other countries decreases.

There are two kinds of fields...

1. Scalar Fields: magnitude but no direction
   - **Example 1**: Heat field from a fire: If you stand by a campfire, you can measure the magnitude (temperature) of the field with a thermometer; if you are close to the fire you will measure a stronger field (higher temperature), but if you move away the field strength decreases (lower temperature). You would not be saying anything about a direction, like “25°C South”.

2. Vector Fields: magnitude and direction
   - **Example 2**: A gravity field is a measure of the acceleration towards the centre of an object.

We deal almost all the time with vector fields in Physics 30.

An electric field exists around any charge (positive or negative).

- If one charge is placed near a second charge, the two fields will “touch” and exert a force on each other.
  - Note: the field is NOT a force, but it does exert a force! It's just like if you are pushing a box; I don’t say you are a force, but you are exerting a force.
  - This meant that physicists had a mathematical way of representing how a force could be transferred over a distance without anything actually touching.

How can we detect and measure the electric fields around charges?

- The easiest way is to place another known charge near by and see how it reacts.
  - We do need to be careful since both the charges have their own fields, so that would affect your results.
Physicists have defined something called a “test charge” as the mathematically perfect charge that could be brought near another charge to measure its electric field.

- It is an infinitely small, positive charge.
- It is a mathematical creation… they don’t really exist.
- Since it is infinitely small, it has a minimal electrical field of its own, so we will treat it as having no electric field.
- It is usually given the symbol q’ (q prime)

Since a test charge is always positive...

- if we see the test charge move towards the other charge we know that one must be negative or
- if the test charge moves away from the other charge, then that one must be positive.

**Measuring Electric Fields**

According to Coulomb’s Law, the force exerted on the test charge must be directly proportional to its own charge and the charge on the other object.

\[ F_e \propto q_1 q_2 \]

substitute the test charge in this relation...

\[ F_e \propto q' q_2 \]

If you divide the force by the charge on the test charge, you get a new formula.

\[ \vec{E} = \frac{F_e}{q'} \]

\( \vec{E} \) = electric field (N/C)

(Arrow above “E” in formula shows this is a vector; it isn’t “energy” which is a scalar)

\( F \) = force (N)

\( q' \) = charge on test charge (C)

**Example 3:** I place a 3.7 C test charge 2.7m to the right of another charge. If there is an attractive force of 2.45N acting on the test charge, **determine** the field strength of the main charge at that location.

We don’t need the distance to figure this question out. It is important to know that the test charge is to the right of the other charge, since we need to give a direction.

\[ \vec{E} = \frac{F_e}{q'} = \frac{2.45}{3.7} = 0.66 \text{ N/C} \quad \text{[left]} \]
The field points left because that’s the direction the test charge is being pulled. **By definition, the direction of an electric field is the direction a positive test charge is pushed or pulled.**

![Diagram of electric field direction](image)

**Figure 2: Direction of electric field near positive and negative charges.**

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**Super Important Note!**

One of the most important things to remember when using this formula is which charge is used. Do you use the charge creating the field, or the charge that is placed nearby to measure the field (the test charge). The answer is, as shown above, the test charge. But this is often something that students forget or mix up. There is a way to remember.

Let’s keep in mind that you’ve already studied fields, you just didn’t necessarily know that it was a field. Gravity. You learned lots of formulas for measuring gravitational fields in Physics 20. The strength of a gravitational field is really a measurement of the acceleration due to gravity in metres per second squared. We can look at the parallels between these two formulas to remember things about each of them.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g = \frac{F}{m}$</td>
<td>$g = \frac{F}{m}$ is the measurement of the <strong>gravitational</strong> field strength. $F$ is the force acting on the object, $m$ is the mass of the small object (like a person), <strong>not</strong> the mass of the large object (like the earth). <em>This formula measures the amount of force per unit mass.</em></td>
</tr>
<tr>
<td>$\vec{E} = \frac{F_e}{q'}$</td>
<td>$\vec{E} = \frac{F_e}{q'}$ is the measurement of the <strong>electric</strong> field strength. $F_e$ is the force acting on the object, $q'$ is the charge of the test charge, <strong>not</strong> the main charge making the electric field. <em>This formula measures the amount of force per unit charge.</em></td>
</tr>
</tbody>
</table>

Just like when you use the formula $F = mg$ you (usually) use a small mass that is sitting on or near a planet that is creating a gravitational field, the charge in the formula $\vec{E} = \frac{F_e}{q'}$ is a small charge sitting near a bigger charge that is making the electric field.

The electric field around a charge will be different at different locations around the charge.

- Further away from the charge, the magnitude of the force will decrease.

$$F_e = \frac{1}{r^2}$$
• The direction will be different.

Example 4: A force of 2.1 N is exerted on a 9.2 e-4 C test charge when it is placed in an electric field created by a 7.5 C charge. If the force is pushing it West, determine the electric field at that point.

\[ \vec{E} = \frac{F_e}{q'} = \frac{2.1}{9.2 \times 10^{-4}} = 2.3 \times 10^3 \text{ N/C [West]} \]

Example 5: If a positive test charge of 3.7e-6 C is put in the same place in the electric field as the original test charge in the last example, determine what force will be exerted on it.

\[ F_e = \vec{E} q' = 2.3 \times 10^3 (3.7 \times 10^{-6}) = 8.4 \times 10^{-3} \text{ N [West]} \]

There is another way to measure electric field strength based on a combination of the formula used above and Coulomb’s Law...

\[ F_e = \frac{k q_1 q_2}{r^2} \quad \vec{E} = \frac{F_e}{q'} \]

• We will change the formula for Coulomb's Law a bit by calling the charges Q (the charge making the field) and q' (the test charge) so the formulas have similar parts.
• Coulomb's Law can now be substituted into the field formula to get…

\[ \vec{E} = \frac{F_e}{q'} = \frac{(k Q q')}{r^2} = \frac{k Q q'}{r^2} \left( \frac{1}{q'} \right) = \frac{kQ}{r^2} \]
• This gives us our new electric field formula:

\[ \vec{E} = \frac{kQ}{r^2} \]

- \( \vec{E} \) = electric field (N/C)
- \( k \) = Coulomb's Constant
- \( Q \) = large charge making the electric field (C)
- \( r \) = distance from the charge (m)

• So, in the formula above, you will use the main big charge that is actually producing the field as “Q”.

• This is great! Now you don’t have to rely on some imaginary thing like a test charge to calculate the field around a regular charge!

**Super Important Note!**

Just as we were able to find a connection between electrostatics and gravity a couple pages back, we can do the same thing with our new formula.

<table>
<thead>
<tr>
<th>( g ) = ( \frac{GM}{r^2} )</th>
<th>( \vec{E} = \frac{kQ}{r^2} )</th>
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<tbody>
<tr>
<td>( g ) = measurement of the gravitational field strength</td>
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<tr>
<td>( G ) = gravitational constant</td>
<td>( k ) = Coulomb's constant</td>
</tr>
<tr>
<td>( M ) = mass of body producing gravitational field</td>
<td>( Q ) = charge of body producing electric field</td>
</tr>
<tr>
<td>( r ) = distance from centre of body</td>
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**Example 6**: A tiny metal ball has a charge of \(-3.0e-6\) C. What is the magnitude and direction of the field at a point, \( P \), 30cm away?

\[ \vec{E} = \frac{kQ}{r^2} = \frac{8.99e9 \times (-3.0e-6)}{0.30^2} = -3.0e5 \text{ N/C} \]

Remember the electric field is always defined as being in the direction that a positive test charge would move. Since the charge producing this field is negative, a positive charge would be attracted towards it. This field points towards the metal ball. That’s the direction you would state.

Get used to names for a particular spot like “P”, since sometimes we may want to relate what you’re doing in a question to several spots, like “P”, “D”, and “A”.
Field Lines

An electric field is a vector.

- Therefore, we can represent an electric field with arrows drawn at various points around an object with charge.
- These electric field lines (sometimes also called lines of force) are drawn below for two simple examples.

![Figure 4: Field lines around a positive object](image1)

![Figure 5: Field lines around a negative object](image2)

Notice that the lines are drawn to show the direction of the force, due to the electric field, as it would act on a positive test charge.

- Also, the closer you get to the charge, the closer the lines are to each other. This symbolizes how the electric field gets stronger as you go closer to the source.
- If you pick a spot further out, you’ll see that the lines aren’t as dense there… so the field is weaker.

What would it look like if you had these two charges close enough to each other that their field lines could interact?

![Figure 6: Two opposite charges showing the interaction of their field lines.](image3)

- The arrows go from the positive charge to the negative charge (in exactly the same direction we would expect a positive test charge to move).
- The direction of the field at any point is the tangent drawn to the field line at that point.
Another important example of field lines comes out of the need to sometimes have a constant, **uniform** electric field.

- As you can see in Figure 6, the field has very different field strengths at different points… it’s irregular.
- That’s because it is made up of only two charges, so the field lines wrap around a lot.
- If we could get a whole bunch of charges lined up evenly then we could get a more uniform electric field.
- It is possible to set this up using two plates that are parallel to each other with opposite charges built up on them.

The field lines are very uniform all the way, except for a slight curvature at the ends.

- We often ignore this slight curvature, since it is very small as long as the plates have a big surface area and are close together. We just make certain not to do any experiments near the ends.
- We can say that we have a constant electric field between these parallel plates.