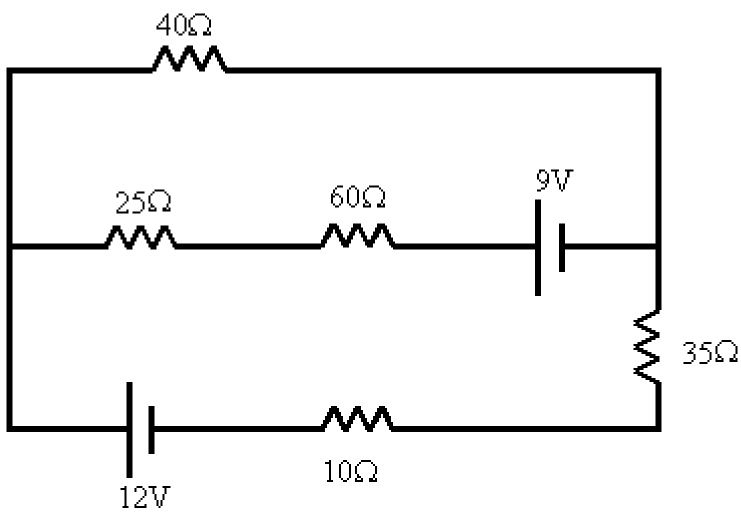
Note-A-Rific: Super Kirchhoff!

It's quite unlikely that you will ever be faced with a problem this difficult on any exam.

- Still, it's important for you to see just how far reaching Kirchhoff's Rules can be, and the way that they can be applied to difficult circuits.
- Kirchhoff really had circuits even more complex than this in mind when he developed his rules.
- Just look at the circuit diagram for a dishwasher and you'll see how complex the circuits can be for an everyday device.

Problem:

Determine the current in each part of this circuit.



Solution:

To figure this out, you will need to also calculate the voltage drops in each part of the circuit.

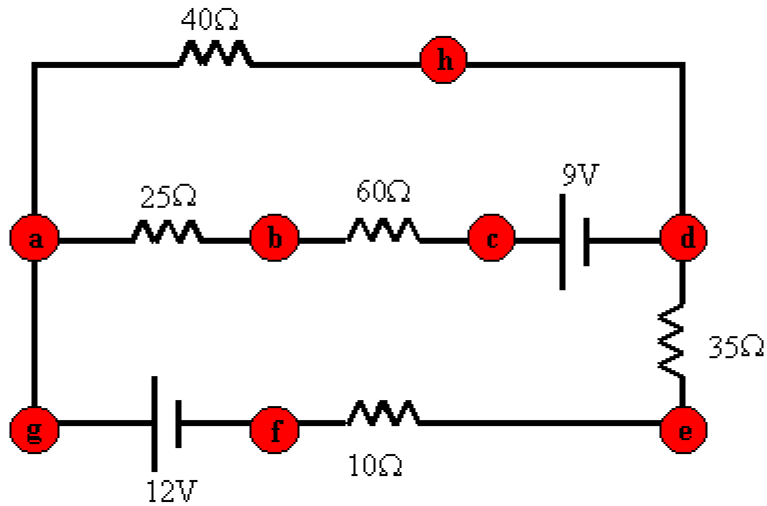
- If you are ever asked to do a similar question where you are asked only for the voltage drops, do that question the same way as this one

.

I've put in some letters to divide up the parts of the circuit where something might happen to the voltage.

- Notice that in between the letters there is only one resistor or one battery… I've isolated the circuit into manageable parts.
- Since we are dealing with resistors (where voltage will drop) and batteries (where voltage will increase), we will need to use + and – to show when voltage has increased or decreased.
- I'm going to need to trace two paths through this circuit, which will overlap on one of the branches.

- You’ll need to keep jumping back and forth between this diagram and the three parts of the solution, so I’ve included some hyperlinks to make it easier to jump back and forth.

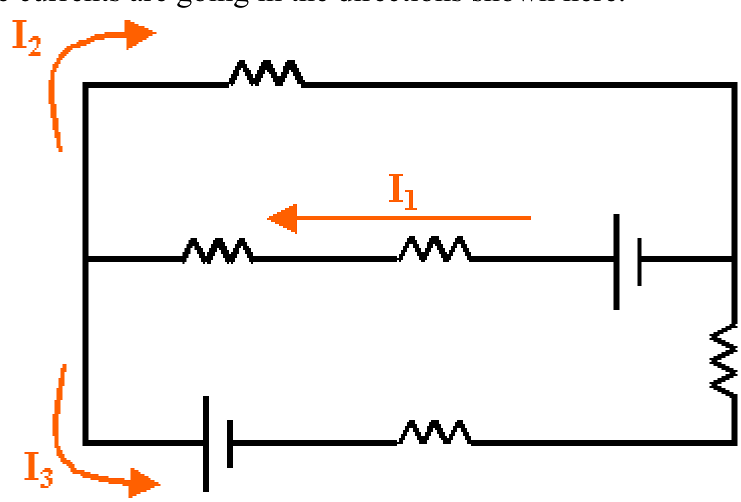


Jump back to…

Part 1 Part 2 Part3

of the solution.

Assume that the currents are going in the directions shown here.



- According to this diagram $I_1 = I_2 + I_3$
- Why did I choose these directions…? Why not!
- You need to start a question like this by arbitrarily choosing some directions for the current flow.
- In the end if we find a value for one of these currents that is negative, it just means we drew the arrow pointing in the wrong direction.
- Now I can start writing down formulas for each section, using formulas like V = IR.
    - Even though I may have some unknowns, I can use the labels drawn above to represent different currents in different branches.

Part 1: Voltage drops along the path [ahdcba]

click here to jump to the diagram

$_aV_h = -I_2\,(40\Omega)$ $\leftarrow$ I put in the minus sign since I will LOSE potential across a resistor

$_hV_d = 0$ V $\leftarrow$ nothin' there!

$_dV_c = -9V$ $\leftarrow$ I'm going the wrong way across the battery for an electron, so it will lose 9 V

$_cV_b = -I_1\,(60\Omega)$

$_bV_a = -I_1\,(25\Omega)$

Since these last two are resistors in series ($_cV_b$ & $_bV_a$), I could write down

$$_cV_a = -I_1\,(60\Omega + 25\Omega)$$
$$_cV_a = -I_1\,(85\Omega)$$

Now we're back to where we started… point "a' on the diagram. Remember, the voltage drops across any complete path along a circuit must equal zero!

$_aV_h + {}_hV_d + {}_dV_c + {}_cV_b + {}_bV_a = 0$

$-I_2\,(40\Omega) + 0 + -9 + -I_1\,(85\Omega) = 0$ (simplify)

$-40\,I_2 + -85\,I_1 + -9 = 0$

Because I (the super smart teacher) know where this problem is going, solve the equation above for I_2 …

- That way I can do some substitution later on into $I_1 = I_2 + I_3$
- You don't always have to do them this way.

$$I_2 = (85I_1 + 9)\,/\,{-40}$$
$$I_2 = -2.125\,I_1 + -0.225$$

click here to jump to the diagram

Part 2: Voltage drops across [agfedcba]

$_aV_g = 0V$

$_gV_f = +12V$ (notice this is the path the electron takes through the battery, so there is a potential increase and we use a + sign)

$_fV_e = -I_3\,(10\Omega)$ → $_fV_d = -I_3\,(45\Omega)$

$_eV_d = -I_3\,(35\Omega)$ →

$_dV_c = -9V$

$_cV_a = -I_1\,(85\Omega)$ $\leftarrow$same as the resistors we added in part 1

$_aV_g + {}_gV_f + {}_fV_c + {}_eV_d + {}_dV_c + {}_cV_a = 0$

$0 + 12 + -I_3\,(45\Omega) + -9 + -I_1\,(85\Omega) = 0$

$-85I_1 + -45\,I_3 + 3 = 0$

Solve for I_3 (again, trust me…)

$I_3 = (85I_1 - 3)\,/\,{-45}$

$I_3 = -1.8889I_1 + 0.06667$

click here to jump to the diagram

Part 3: Solve $I_1 = I_2 + I_3$

Substitute in the values you found above for I_2 and I_3

$I_1 = I_2 + I_3$

$= (-2.125\ I_1 + -0.225) + (-1.8889I_1 + 0.06667)$

$I_1 = -4.014I_1 + -0.1583$

$5.014I_1 = -0.1583$

$I_1 = -0.032$ A

- The minus sign means we drew our arrow for I_1 in the wrong direction.

Substitute the value you just got for I_1 into the relationship for I_2.

$I_2 = -2.125\ I_1 + -0.225 = -0.16$A

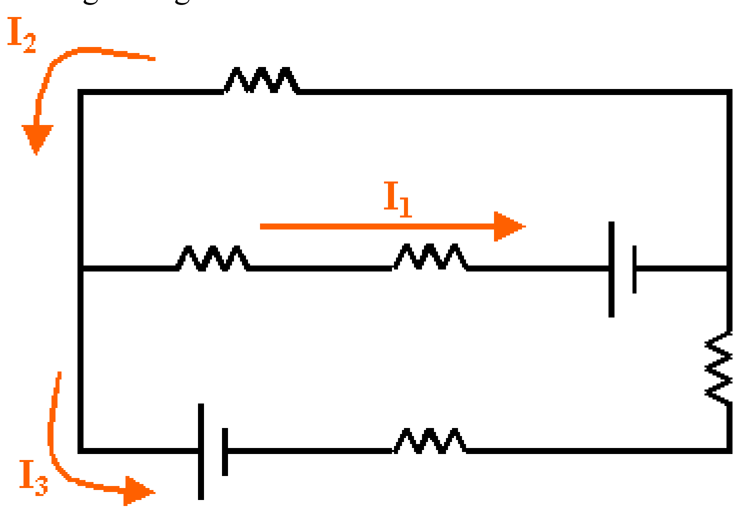
- The minus sign means we drew our arrow for I_2 in the wrong direction.

Substitute the value you got for I_1 into the relationship for I_3.

$I_3 = -1.8I_1 + 0.6 = 0.13$A

click here to jump to the diagram

So the currents flowing through the circuit would look like this.



Since you now know the currents in each part, you could also calculate the voltage drops in each part using V =IR.

- Go ahead and try… the answers are:
  - ${}_aV_h = 6.4$V
  - ${}_cV_b = 1.92$V
  - ${}_bV_a = 0.8$V
  - ${}_fV_e = 1.3$V
  - ${}_eV_d = 4.55$V