Lesson 3: Conservation of Momentum in 1-D Collisions

A collision is when two or more objects strike each other, and exert a relatively large force during a relatively short period of time.

- This force acting during a time period results in impulse.
- We can look at what is happening based on Newton's Third Law, just like we looked at Newton's Second Law for impulse.
  - Imagine that two balls of **unequal masses** traveling at **different velocities** ram into each other in a head on collision, as shown below…

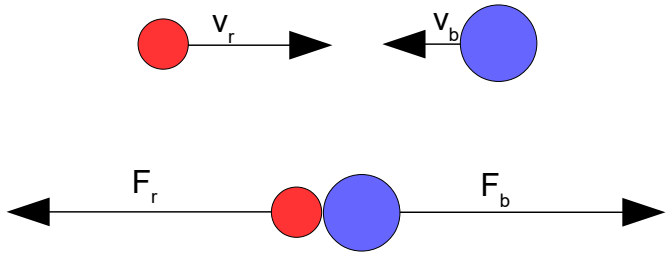

Illustration 1: Collision of a red and blue ball, showing initial velocities, and then forces during collision.

  - Notice that during the collision, according to Newton's Third Law, the balls exert **equal**, but **opposite forces** on each other.
  - If the two forces are equal but opposite, they must add up to zero.

$$0 = F_r + F_b$$
$$0 = F_r \Delta t + F_b \Delta t$$
$$0 = \Delta p_r + \Delta p_b$$
$$0 = (p_r' - p_r) + (p_b' - p_b)$$
$$0 = (m_r v_r' - m_r v_r) + (m_b v_b' - m_b v_b)$$
$$0 = m_r v_r' - m_r v_r + m_b v_b' - m_b v_b$$
$$m_r v_r + m_b v_b = m_r v_r' + m_b v_b'$$

At first we just show that the two forces add up to zero.
Then we multiply everything by Δt (note that zero times anything is still zero).
Remember that FΔt is impulse, Δp, so replace it.
Δp = p' – p , where the ' symbol is "prime" and just means final.
p = mv, so we replace that too.
After dropping the brackets, we move everything around.

The Law of Conservation of Momentum remains a fundamental law of physics. Like all conservation laws, it essentially means whatever you started with you still have at the end.

- The formula manipulation shown here will work for problems with two objects in a head-on collision, called either a ***1-dimensional*** (1-D) or ***linear*** collision.
- The objects must move in a straight line... they can ***not*** move off at any sort of angle.

Conservation of Momentum is true if the objects are acting in an isolated system, where no matter is entering or leaving, and the energy remains constant. This means that there are no external forces acting on the objects.

Isolated: No **matter** or **energy** is allowed to enter or leave the system.
Closed: No **matter** is allowed to enter or leave the system. **Energy** can enter or leave.
Open: **Energy** and **matter** can enter or leave.

Warning
It is critical that you understand that you can only use the conservation of momentum in **isolated systems**. Non-conservative forces such as fricition can ***NOT*** be acting on any of the objects.

You do have to be careful with how you solve these collision problems using conservation of momentum.

- After the collision the two objects might bounce apart (**Example 1**)...
- ...or the objects might stick together (**Example 2**).

Example 1: Objects bounce apart

A 0.15kg blue ball moving at 8.0m/s to the right hits a 0.10 kg red billiard ball at rest. If the blue ball continues to move to the right at 2.5m/s, **determine** the velocity of the red ball.

$$p_{total} = p_{total}'$$
$$p_b + p_r = p_b' + p_r'$$
$$m_b v_b + m_r v_r = m_b v_b' + m_r v_r'$$
$$0.15\ (8.0) + 0.10\ (0) = 0.15\ (2.5) + 0.10\ (v_r')$$
$$v_r' = 8.25 = 8.3\ \text{m/s [right]}$$

Example 2: Objects stick together

Two balls of clay, a blue one being 2.3kg and the second red one being 5.6kg, hit each other and stick together. If the blue one was moving to the right at 12m/s, and the red was moving at 8.1m/s to the left, **determine** their final velocity.

$$p_{total} = p_{total}'$$
$$p_b + p_r = p_b' + p_r'$$
$$m_b v_b + m_r v_r = m_b v_b' + m_r v_r'$$
$$m_b v_b + m_r v_r = v'\ (m_b + m_r)$$
$$2.3\ (+12) + 5.6\ (-8.1) = v`\ (2.3 + 5.6)$$
$$v' = -2.2481 = -\ 2.2\ \text{m/s}\ \ \textbf{or}\ \ 2.2\ \text{m/s [left]}$$

Since the two lumps are stuck together, you add the masses together after the collision. Also, since they are one big lump now, they must have the same velocity, so you only have v'.

Example 2 (sticking together) showed a situation where the two objects stick together after hitting each other.

- This is a very common sort of question, since it could involve objects like two train cars colliding and then locking together afterwards.
- It is also possible for two objects to be stuck together at the start, and then go apart afterwards.
  - If this happens you'd just have to reverse the left and right hand sides of the formula.

Homework

p476 #2
p477 #2
p478 #1
p479 #2